M_e^v-Polynomial and Vertex-Edge Topological Indices of Graphs

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ABSTRACT

Let G be a simple connected undirected graph with n vertices and m edges, the vertex to edge version of M-polynomial is a graph polynomial based on the comparison between vertices and edges of the graph G. We denote the vertex to edge version of M-polynomial by M_e^v-Polynomial and defined by: $M_e^v(G,x,y) = \sum_{i=1}^{n} \sum_{j=1}^{d_G(v_i)} x^{d_G(v_i)} y^{d_G(e_j)}$ where $d_G(v_i)$ are degrees of vertices v_i and $d_G(e_j)$ are degrees of edges e_j which are defined to be the number of neighbors of e_j. In this paper, the M_e^v-polynomial and two new proposed vertex-edge version of topological indices, namely vertex-edge versions of geometric arithmetic GA_e^v, and Nirmala N_e^v indices have been calculated for selecting graphs.

1. Introduction

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$ having n vertices and m edges. Let $\mathcal{E}$ denote the set of all graphs and $R[x_1, x_2, \ldots, x_n]$ be the ring of polynomials in the variables $x_1, x_2, \ldots, x_n$. A graph polynomial $P(G, x_1, x_2, \ldots, x_n)$ is a function from $\mathcal{E}$ to $R[x_1, x_2, \ldots x_n]$ [1]. A large number of graph polynomials have been introduced in the literature, several of which have found applications in the field of mathematical chemistry. For example, the well-known distance-based polynomial is Hosoya polynomial [2], M-polynomial and CoM-polynomial [3–5].

The defined polynomials are based on different categories; we see Hosoya polynomial based on distance between vertices while the M-polynomial [3] based on degree of vertices. one of the significant of graph polynomials is to obtain some topological indices from them, as the Wiener index is the first derivative of Hosoya polynomial at $x = 1$ [6], while almost all degree based topological indices can be obtained from the M-polynomial [7–10]. M-polynomial can be count as an important graph polynomial in graph theory, due to its significant this concept has been extended to define new polynomials. The CoM-polynomial, for example, is considered for non-adjacent vertices, that yields degree-based topological coindices [11]; the Neighborhood M-polynomial which considered as a neighborhood degree-based polynomial [12]; and the M_{ve}-polynomial which based on vertex-edge degree of vertices of the graph [13].

The main goal of this paper is to establish a new version of M-polynomial with derivation of two new vertex-edge comparison topological indices from the proposed polynomial. Additionally, all results obtained for certain graphs.

2. M_e^v-polynomial and ve-topological index

2.1. Definition Let G be a simple graph with n vertices and m edges the vertex to edge version of the M-polynomial is a graph polynomial based on the comparison between vertices and edges of the graph G. We denote the vertex to edge version of M-polynomial by M_e^v-Polynomial and defined by:

$$M_e^v(G,x,y) = \sum_{i=1}^{n} \sum_{j=1}^{d_G(v_i)} x^{d_G(v_i)} y^{d_G(e_j)} \tag{1}$$

where $d_G(v)$ is the degree of the vertex v and $d_G(e)$ is the degree of the edge e which is defined to be the number of neighbors of e, that is if $e = vw$, then $d_G(e) = |N(v) \cup N(w)| - 2$.

Unlike the classical M-polynomial, which is defined through adjacency of vertices and thus encodes only information about direct vertex connections, the M_e^v-Polynomial incorporates the incidence structure between vertices and edges. This polynomial is needed because it provides a compact algebraic tool that simultaneously encodes information about both vertices and edges of a graph. This dual perspective allows us to capture structural properties that cannot be fully described by M-polynomial and others such as CoM-polynomial, NM-polynomial and M_{ve}-polynomial.

We can bring a simple graph such as complete graph K_4 and apply Definition 2.1 to find $M_e^v(K_4,x,y)$ as follows: First, we see K_4 has 4 vertices with degree 3 and 6 edges with degree 4. so $M_e^v(K_4,x,y) = \sum_{i=1}^{4} \sum_{j=1}^{3} x^{d_{K_4}(v_i)} y^{d_{K_4}(e_j)} = \sum_{i=1}^{4} \sum_{j=1}^{3} x^3 y^4 = 4x^3(y^4 + y^4 + y^4) = 12x^3y^4$.

A vertex-edge topological index (resp. *ve*-topological index) of a given graph $G = (V,E)$ is a graph invariant of the form

$$I_e^v(G) = \sum_{ve} f(d_G(v), d_G(e)) \tag{2}$$

where $I_e^v(G)$ stands for general form of *ve*-topological index, ve indicates the vertex v and edge e are incident in G and the function f gives the formula of the index.
If the general form of degree-based topological indices is given by

$$I(G) = \sum_{vw \in E(G)} f(d_G(v), d_G(w)) \tag{3}$$

[3], then Equation 2 represents it's corresponding vertex-edge version.
The vertex-edge version of two well-known topological indices, namely Geometric Arithmetic (GA) [14], and Nirmala [4,15] with their computation formulas will be presented in the following table:

Table 1: Some *ve*-topological indices with their formula of computing from $M_e^v(G,x,y)$

$ve-$ indices	$f(d_G(v), d_G(e))$	derivation from $M_e^v(G,x,y)$
$ve-$ GA index	$GA_e^v(G) = \sum_{ve} \frac{2\sqrt{d_G(v)d_G(e)}}{d_G(v) + d_G(e)}$	$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}[M_e^v(G,x,y)]_{x=1}$
$ve-$ nirmala index	$N_e^v(G) = \sum_{ve}\sqrt{d_G(v) + d_G(e)}$	$\left(D_x^{\frac{1}{2}}J\right)[M_e^v(G,x,y)]\Big\vert_{x=1}$

Where $GA_e^v(G)$, $N_e^v(G)$ refer to the ve-versions of geometric arithmetic and nirmala indices of the graph G. Also used operators are defined as [3,4,7,10]:

$$S_x = \int_0^x \frac{M_e^v(G,z,y)}{z} dz,\ J(M_e^v(G,x,y)) = M_e^v(G,x,x), D_x^{1/2}(M_e^v(G,x,y)) = \sqrt{x \frac{\partial(M_e^v(G,x,y))}{\partial x}} \sqrt{M_e^v(G,x,y)},$$

$$D_y^{1/2}(M_e^v(G,x,y)) = \sqrt{y \frac{\partial(M_e^v(G,x,y))}{\partial y}} \sqrt{M_e^v(G,x,y)}.$$

Topological indices are essential as they provide simple numerical values that describe the structure of molecules or networks. Without the need for complex experiments, these values help scientists in predicting the physical, chemical, and biological characteristics of compounds. They are effective tools for connecting structure and function together in a variety of domains, including network analysis, materials science, and drug design [4,9,10]. The friendship graph $\mathcal{F}_n$, also known as the Dutch windmill graph, is formed by joining n triangles at a common vertex and has applications in social network analysis and Ramsey theory [16]. The sun graph is a cycle with additional pendant vertices, useful in modeling symmetric but extended structures [17]. Likewise, the helm graph and its generalization-the generalized helm graph-arise by attaching pendant edges to the vertices of a cycle, and they possess notable applications in chemical graph theory and communication topology [16,18]. The shell graph [19]. A shell graph and a path P_2 graph are combined to create an ice-cream graph, where $n > 3$ share a common end point known as the apex vertex v_2 and v_{n-1} remain the same. It is denoted by I_n and models a cone-like structure built on a central hub with layered cycles [20].

The main objective of this work is to derive the M_e^v-polynomial for each considered graphs. The practical relevance of this objective lies in the fact that this polynomial can be seen as a generating function from which a wide range of ve-topological indices can be directly computed. In the sections that follow, we present new results related to M_e^v-polynomials with some of ve-indices for the above-mentioned special graphs. These results not only extend earlier studies but also point to structural patterns that may direct future theoretical developments and applications.

3. M_e^v-polynomial of Certain graphs

3.1 Theorem Let $\hat{S}_n$ be the Shell graph, for all $n \geq 4$ then M_e^v-polynomial of $\hat{S}_n$ is

$$M_e^v(\hat{S}_n, x, y) = (xy)^{n-1}[(n-3)y+2] + 2x^2y^3[y^{n-4}+1] + (xy)^3[(n-3)y^{n-3} + 2(n-4)y + 2] \quad (4)$$

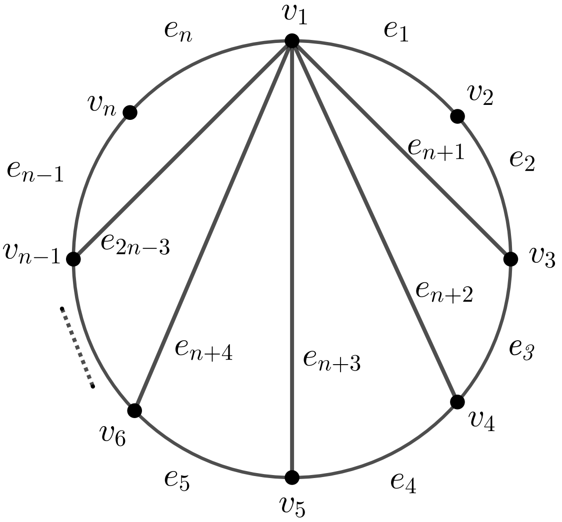


Figure 1: Shell Graph $\hat{S}_n$

Proof:

We see that, the shell graph has *n* vertices and *2n-3* edges with $d_{\hat{S}_n}(v_1) = n-1, d_{\hat{S}_n}(v_2) = d_{\hat{S}_n}(v_n) = 2$ *and* $d_{\hat{S}_n}(v_3) = d_{\hat{S}_n}(v_4) = \ldots = d_{\hat{S}_n}(v_{n-1}) = 3$. Also, $d_{\hat{S}_n}(e_1) = d_{\hat{S}_n}(e_n) = n-1$, , $d_{\hat{S}_n}(e_2) = d_{\hat{S}_n}(e_{n-1}) = n$, , $d_{\hat{S}_n}(e_3) = d_{\hat{S}_n}(e_4) = \ldots = d_{\hat{S}_n}(e_{n-2}) = 4$ and $d_{\hat{S}_n}(e_{n+1}) = d_{\hat{S}_n}(e_{n+2}) = \ldots = d_{\hat{S}_n}(e_{3n-3}) = n$. Using the above information and apply Definition 2.1 give the following result,

$$\begin{aligned} M_e^v(\hat{S}_n, x, y) &= \sum_{i=1}^{n} \sum_{j=1}^{d_{\hat{S}_n}(v_i)} x^{d_{\hat{S}_n}(v_i)} y^{d_{\hat{S}_n}(e_j)} \\ &= x^{n-1}[2y^{n-1} + (n-3)y^n] + 2x^2(y^3 + y^{n-1}) + 2x^3(y^3 + y^4 + y^n) + (n-5)x^3[2y^4 + y^n] \\ &= (xy)^{n-1}[(n-3)y+2] + 2x^2y^3[y^{n-4}+1] + (xy)^3[(n-3)y^{n-3} + 2(n-4)y + 2]. \end{aligned}$$

3.2 Theorem Let I_n be the Ice-cream graph, for all $n \geq 4$ then M_e^v-polynomial of I_n is

$$M_e^v(I_n, x, y) = (n-1)x^{n-1}y^n + x^3[(n-1)y^n + 2(n-2)y^4 + 2y^3] + 2x^2y^3 \qquad (5)$$

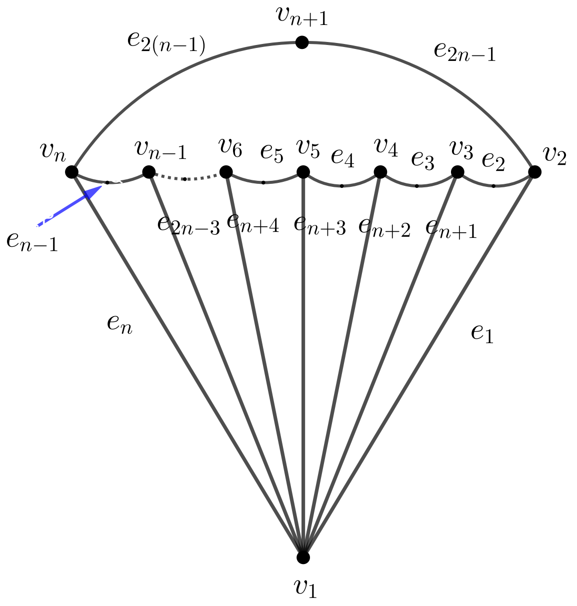


Figure 2: Ice-cream Graph I_n

Proof:

We see that, the Ice-cream graph has *n+1* vertices and 2(*n*-1) edges with $d_{I_n}(v_1) = n - 1, d_{I_n}(v_2) = d_{I_n}(v_3) = \ldots = d_{I_n}(v_n) = 3$ *and* $d_{I_n}(v_{n+1}) = 2$. Also, $d_{I_n}(e_1) = d_{I_n}(e_{n+1}) = \ldots = d_{I_n}(e_{2n-3}) = d_{I_n}(e_n) = n,$, $d_{I_n}(e_2) = d_{I_n}(e_3) = \ldots = d_{I_n}(e_{n-1}) = 4$, and $d_{I_n}(e_{2n-1}) = d_{I_n}(e_{2(n-1)}) = 3$. Using the above information and apply Definition 2.1 give the following result,

$$\begin{aligned} M_e^v(I_n, x, y) &= \sum_{i=1}^{n+1} \sum_{j=1}^{d_{I_n}(v_i)} x^{d_{I_n}(v_i)} y^{d_{I_n}(e_j)} \\ &= (n-1)x^{n-1}y^n + 2x^3(y^n + y^4 + y^3) + (n-3)x^3(y^n + 2y^4) + 2x^2y^3 \\ &= (n-1)x^{n-1}y^n + x^3[(n-1)y^n + 2(n-2)y^4 + 2y^3] + 2x^2y^3. \end{aligned}$$

3.3 Theorem Let $\mathcal{F}_n$ be the Friendship graph, for all $n \geq 2$ then M_e^v-polynomial of $\mathcal{F}_n$ is

$$M_e^v(\mathcal{F}_n, x, y) = 2n(xy)^2\left[(xy)^{2(n-1)} + y^{2(n-1)} + 1\right] \qquad (6)$$

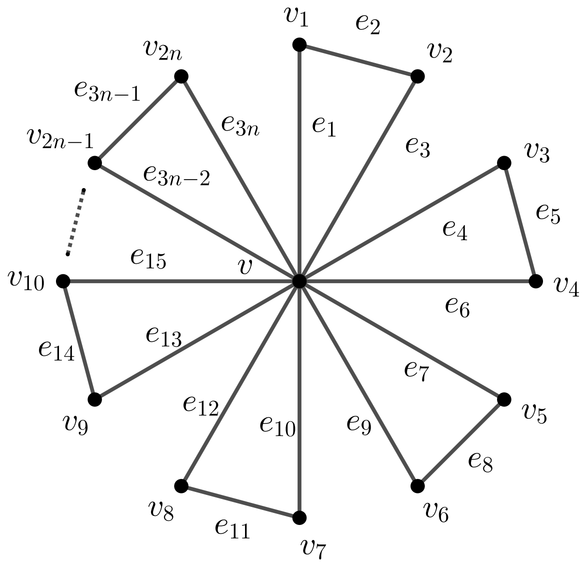


Figure 3: Friendship Graph $\mathcal{F}_n$

Proof:

We see that, the Friendship graph has $2n$ vertices and $3n$ edges with $d_{\mathcal{F}_n}(v_1) = d_{\mathcal{F}_n}(v_2) = \ldots = d_{\mathcal{F}_n}(v_{2n}) = 2$ *and* $d_{\mathcal{F}_n}(v) = 2n$. Also, $d_{\mathcal{F}_n}(e_{3i-1}) = 2$, for all i=1,2,3,...,n and $d_{\mathcal{F}_n}(e_{3i-2}) = d_{\mathcal{F}_n}(e_{3i}) = 2n$, for all i=1,2,3,...,n. Using the above information and apply Definition 2.1 give the following result,

$$\begin{aligned} M_e^v(\mathcal{F}_n, x, y) &= \sum_{i=1}^{2n+1} \sum_{j=1}^{d_{\mathcal{F}_n}(v_i)} x^{d_{\mathcal{F}_n}(v_i)} y^{d_{\mathcal{F}_n}(e_j)} \\ &= 2nx^{2n}y^{2n} + 2nx^2(y^2 + y^{2n}) \\ &= 2n(xy)^2\left[(xy)^{2(n-1)} + y^{2(n-1)} + 1\right]. \end{aligned}$$

3.4 Theorem Let $\mathcal{S}_n$ be the Sun graph, for all $n \geq 3$ then M_e^v-polynomial of $\mathcal{S}_n$ is

$$M_e^v(\mathcal{S}_n, x, y) = nxy^2[2(xy)^2 + x^2 + 1] \tag{7}$$

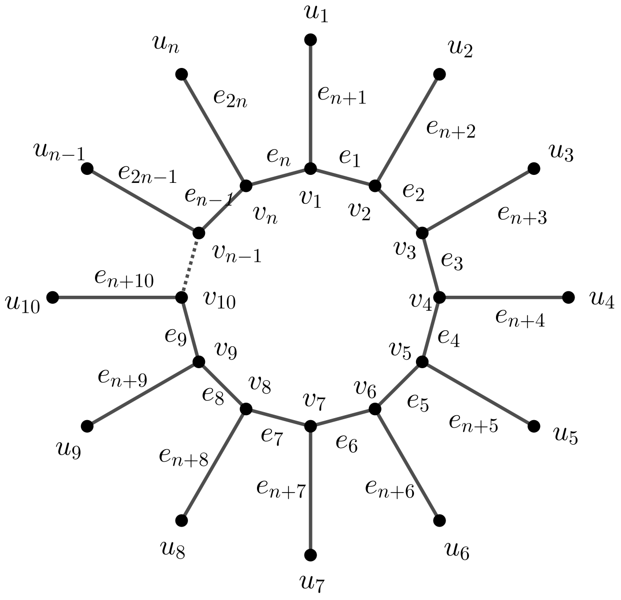


Figure 4: Sun Graph $\mathcal{S}_n$

Proof:

We see that, the Sun graph has $2n$ vertices and $2n$ edges with $d_{\mathcal{S}_n}(v_1) = d_{\mathcal{S}_n}(v_2) = \ldots = d_{\mathcal{S}_n}(v_n) = 3$ *and* $d_{\mathcal{S}_n}(u_1) = d_{\mathcal{S}_n}(u_2) = \ldots = d_{\mathcal{S}_n}(u_n) = 1$ Also, $d_{\mathcal{S}_n}(e_1) = d_{\mathcal{S}_n}(e_2) = \ldots = d_{\mathcal{S}_n}(e_n) = 4$ and $d_{\mathcal{S}_n}(e_{n+1}) = d_{\mathcal{S}_n}(e_{n+2}) = \ldots = d_{\mathcal{S}_n}(e_{2n}) = 2$. Using the above information and apply Definition 2.1 give the following result,

$$\begin{aligned} M_e^v(\mathcal{S}_n, x, y) &= \sum_{i=1}^{2n} \sum_{j=1}^{d_{\mathcal{S}_n}(v_i)} x^{d_{\mathcal{S}_n}(v_i)} y^{d_{\mathcal{S}_n}(e_j)} \\ &= nx^3(2y^4 + y^2) + nxy^2 \\ &= nxy^2[2(xy)^2 + x^2 + 1]. \end{aligned}$$

3.5 Theorem Let $\mathcal{H}_n$ be the Helm graph, for all $n \geq 3$ then M_e^v-polynomial of $\mathcal{H}_n$ is

$$M_e^v(\mathcal{H}_n, x, y) = nxy^3[(xy)^{n-1} + x^3y^{n-1} + 2(xy)^3 + x^3 + 1] \tag{8}$$

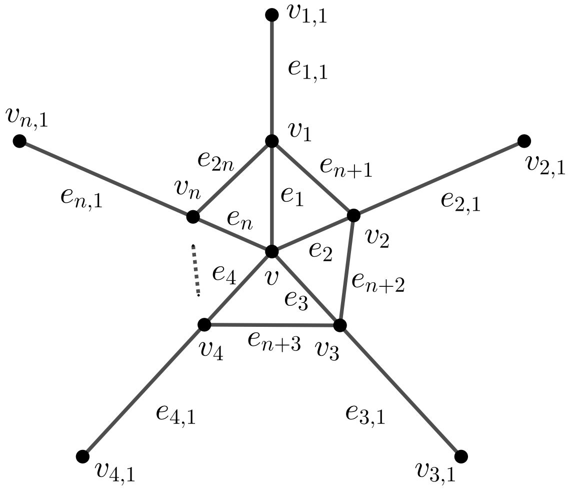


Figure 5: Helm Graph $\mathcal{H}_n$

Proof:

We see that, the Helm graph has $2n+1$ vertices and $3n$ edges with $d_{\mathcal{H}_n}(v_1) = d_{\mathcal{H}_n}(v_2) = \ldots = d_{\mathcal{H}_n}(v_{2n}) = 4$, $d_{\mathcal{H}_n}(v_{i,1}) = 1$ for all i=1,2,3,...,n *and* $d_{\mathcal{H}_n}(v) = n$. Also, $d_{\mathcal{H}_n}(e_1) = d_{\mathcal{H}_n}(e_2) = \ldots = d_{\mathcal{H}_n}(e_n) = n+2$, $d_{\mathcal{H}_n}(e_{n+1}) = d_{\mathcal{H}_n}(e_{n+2}) = \ldots = d_{\mathcal{H}_n}(e_{2n}) = 6$ and $d_{\mathcal{H}_n}(e_{i,1}) = 3$, for all i=1,2,3,...,n. Using the above information and apply Definition 2.1 give the following result,

$$\begin{aligned} M_e^v(\mathcal{H}_n, x, y) &= \sum_{i=1}^{2n+1} \sum_{j=1}^{d_{\mathcal{H}_n}(v_i)} x^{d_{\mathcal{H}_n}(v_i)} y^{d_{\mathcal{H}_n}(e_j)} \\ &= nx^n y^{n+2} + nx^4(y^{n+2} + 2y^6 + y^3) + nxy^3 \\ &= nxy^3[(xy)^{n-1} + x^3y^{n-1} + 2(xy)^3 + x^3 + 1]. \end{aligned}$$

1.6 Theorem Let $\mathcal{H}_n^*$ be the generalized Helm graph, for all $n \geq 3$ then M_e^v-polynomial of $\mathcal{H}_n^*$ is

$$M_e^v(\mathcal{H}_n^*, x, y) = n(xy)^2[x^{n-2}y^n + x^2y^n + 2x^2y^4 + (xy)^2 + y^2 + 2(n-2)] + nx^2y + nxy \quad (9)$$

Proof:

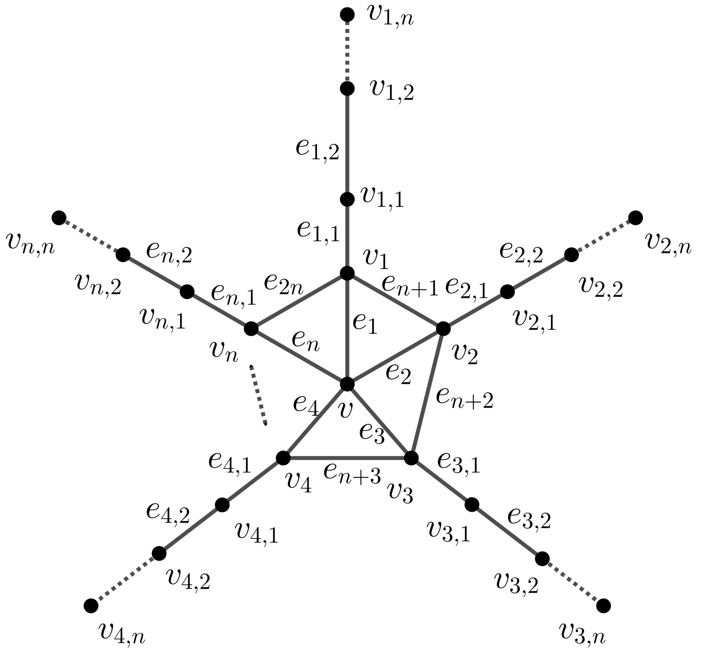
We see that, the generalized helm graph has $n(n+1)+1$ vertices and $n(n+2)$ edges with $d_{\mathcal{H}_n^*}(v_1) = d_{\mathcal{H}_n^*}(v_2) = \ldots = d_{\mathcal{H}_n^*}(v_n) = 4$, $d_{\mathcal{H}_n^*}(v_{i,j}) = 2$ for all i=1,2,3,...,n *and* j=1,2,3,...,n-1, $d_{\mathcal{H}_n^*}(v_{i,n}) = 1$ for all i=1,2,3,...,n *and* $d_{\mathcal{H}_n^*}(v) = n$. Also, $d_{\mathcal{H}_n^*}(e_1) = d_{\mathcal{H}_n^*}(e_2) = \ldots = d_{\mathcal{H}_n^*}(e_n) = n+2$, $d_{\mathcal{H}_n^*}(e_{n+1}) = d_{\mathcal{H}_n}(e_{n+2}) = \ldots = d_{\mathcal{H}_n}(e_{2n}) = 6$,

$d_{\mathcal{H}_n}(e_{i,1}) = 4$, for all i=1,2,3,…,n, $d_{\mathcal{H}_n}(e_{i,j}) = 2$, for all i=1,2,3,…,n and j=2,3,4,…,n-1, *and* $d_{\mathcal{H}_n^*}(e_{i,n}) = 1$ for all i=1,2,3,…,n.

Using the above information and apply Definition 2.1 give the following result,

$$M_e^v(\mathcal{H}_n^*, x, y) = \sum_{i=1}^{n(m+1)+1} \sum_{j=1}^{d_{\mathcal{H}_n^*}(v_i)} x^{d_{\mathcal{H}_n^*}(v_i)} y^{d_{\mathcal{H}_n^*}(e_j)}$$

$$= nx^n y^{n+2} + nx^4(y^{n+2} + 2y^6 + y^4) + nx^2(y^4 + y^2) + 2n(n-3)(xy)^2 + n(xy)^2 + nx^2y + nxy$$



$$= n(xy)^2[x^{n-2}y^n + x^2y^n + 2x^2y^4 + (xy)^2 + y^2 + 2(n-2)] + nx^2y + nxy.$$

Figure 6: Generalized Helm Graph $\mathcal{H}_n^*$

4. ve-topological indices of graphs

In this section, the discussed *ve*-topological indices will be calculated for the presented graphs via M_e^v-polynomial, by using the given operations.

4.1 Corollary

1. $GA_e^v(\hat{\mathcal{S}}_n) = 2\left[\frac{(n-3)\sqrt{n(n-1)}}{2n-1} + \frac{2\sqrt{2(n-1)}}{n+1} + \frac{(n-3)\sqrt{3n}}{n+3} + \frac{4(n-4)\sqrt{3}}{7} + \frac{2\sqrt{6}}{5} + 2\right]$
2. $GA_e^v(I_n) = 2\left[\frac{(n-1)\sqrt{n(n-1)}}{2n-1} + \frac{(n-1)\sqrt{3n}}{n+3} + \frac{4(n-2)\sqrt{3}}{7} + \frac{2\sqrt{6}}{5} + 1\right]$
3. $GA_e^v(\mathcal{F}_n) = 4n\left[\frac{\sqrt{n}}{n+1} + 1\right]$
4. $GA_e^v(\mathcal{S}_n) = 2n\left[\frac{4\sqrt{3}}{7} + \frac{\sqrt{6}}{5} + \frac{\sqrt{2}}{3}\right]$
5. $GA_e^v(\mathcal{H}_n) = 2n\left[\frac{\sqrt{n(n+2)}}{2(n+1)} + \frac{2(n+2)}{n+6} + \frac{2\sqrt{6}}{5} + \frac{15\sqrt{3}}{28}\right]$
6. $GA_e^v(\mathcal{H}_n^*) = 2n\left[\frac{\sqrt{n(n+2)}}{2(n+1)} + \frac{2\sqrt{n+2}}{n+6} + \frac{2\sqrt{6}}{5} + \frac{2\sqrt{2}}{3} + n - 1\right]$

Proof:

1. From Theorem 3.1,

$$M_e^v(\hat{S}_n, x, y) = x^{n-1}[2y^{n-1} + (n-3)y^n] + 2x^2(y^3 + y^{n-1}) + 2x^3(y^3 + y^4 + y^n) \\ +(n-5)x^3[2y^4 + y^n]$$

$$D_y^{\frac{1}{2}}M_e^v(\hat{S}_n, x, y) = 2\sqrt{n-1}(xy)^{n-1} + (n-3)\sqrt{n}x^{n-1}y^n + 2\sqrt{n-1}x^2y^{n-1} \\ +(n-3)\sqrt{n}x^3y^n + 4(n-4)x^3y^4 + 2\sqrt{3}(xy)^3 + 2\sqrt{3}x^2y^3$$

$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\hat{S}_n, x, y) = 2(n-1)(xy)^{n-1} + (n-3)\sqrt{n(n-1)}x^{n-1}y^n + 2\sqrt{2(n-1)}x^2y^{n-1} \\ +(n-3)\sqrt{3n}x^3y^n + 4(n-4)\sqrt{3}x^3y^4 + 6(xy)^3 + 2\sqrt{6}x^2y^3$$

$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\hat{S}_n, x, y) = 2(n-1)x^{2(n-1)} + (n-3)\sqrt{n(n-1)}x^{2n-1} + 2\sqrt{2(n-1)}x^{n+1} \\ +(n-3)\sqrt{3n}x^{n+3} + 4(n-4)\sqrt{3}x^7 + 6x^6 + 2\sqrt{6}x^5$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\hat{S}_n, x, y) = 2\left[x^{2(n-1)} + \frac{(n-3)\sqrt{n(n-1)}}{2n-1}x^{2n-1} + \frac{2\sqrt{2(n-1)}}{n+1}x^{n+1} \right. \\ \left. + \frac{(n-3)\sqrt{3n}}{n+3}x^{n+3} + \frac{4(n-4)\sqrt{3}}{7}x^7 + x^6 + \frac{2\sqrt{6}}{5}x^5\right]$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\hat{S}_n, x, y)_{x=1} = 2\left[\frac{(n-3)\sqrt{n(n-1)}}{2n-1} + \frac{2\sqrt{2(n-1)}}{n+1} + \frac{(n-3)\sqrt{3n}}{n+3} + \frac{4(n-4)\sqrt{3}}{7} \right. \\ \left. + \frac{2\sqrt{6}}{5} + 2\right]$$

It follows from Table 1, $GA_e^v(\hat{S}_n) = 2\left[\frac{(n-3)\sqrt{n(n-1)}}{2n-1} + \frac{2\sqrt{2(n-1)}}{n+1} + \frac{(n-3)\sqrt{3n}}{n+3} + \frac{4(n-4)\sqrt{3}}{7} + \frac{2\sqrt{6}}{5} + 2\right]$.

2. From Theorem 3.2, $M_e^v(I_n,x,y) = (n-1)x^{n-1}y^n + x^3[(n-1)y^n + 2(n-2)y^4 + 2y^3] + 2x^2y^3$ $D_y^{\frac{1}{2}}M_e^v(I_n,x,y) = (n-1)\sqrt{n}x^{n-1}y^n + (n-1)\sqrt{n}x^3y^n + 4(n-2)x^3y^4 + 2\sqrt{3}(xy)^3 + 2\sqrt{3}x^2y^3$

$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(I_n,x,y) = (n-1)\sqrt{n(n-1)}x^{n-1}y^n + (n-1)\sqrt{3n}x^3y^n + 4(n-2)\sqrt{3}x^3y^4 + 6(xy)^3 + 2\sqrt{6}x^2y^3$$

$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(I_n,x,y) = (n-1)\sqrt{n(n-1)}x^{2n-1} + (n-1)\sqrt{3n}x^{n+3} + 4(n-2)\sqrt{3}x^7 + 6x^6 + 2\sqrt{6}x^5$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(I_n,x,y) = 2\left[\frac{(n-1)\sqrt{n(n-1)}}{2n-1}x^{2n-1} + \frac{(n-1)\sqrt{3n}}{n+3}x^{n+3} + \frac{4(n-2)\sqrt{3}}{7}x^7 + x^6 + \frac{2\sqrt{6}}{5}x^5\right]$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(I_n,x,y)_{x=1} = 2\left[\frac{(n-1)\sqrt{n(n-1)}}{2n-1} + \frac{(n-1)\sqrt{3n}}{n+3} + \frac{4(n-2)\sqrt{3}}{7} + \frac{2\sqrt{6}}{5} + 1\right]$$

It follows from Table1, $GA_e^v(I_n) = 2\left[\frac{(n-1)\sqrt{n(n-1)}}{2n-1} + \frac{(n-1)\sqrt{3n}}{n+3} + \frac{4(n-2)\sqrt{3}}{7} + \frac{2\sqrt{6}}{5} + 1\right]$.

3. From Theorem 3.3,

$$M_e^v(\mathcal{F}_n,x,y) = 2n(xy)^2\left[(xy)^{2(n-1)} + y^{2(n-1)} + 1\right]$$

$$D_y^{\frac{1}{2}}M_e^v(\mathcal{F}_n,x,y) = 2n\sqrt{2n}(xy)^{2n} + 2n\sqrt{2n}x^2y^{2n} + 2n\sqrt{2}(xy)^2$$

$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{F}_n,x,y) = 4n^2(xy)^{2n} + 4n\sqrt{n}x^2y^{2n} + 4n(xy)^2$$

$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{F}_n,x,y) = 4n^2x^{4n} + 4n\sqrt{n}x^{2(n+1)} + 4nx^4$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{F}_n,x,y) = 2\left[nx^{4n} + \frac{2n\sqrt{n}}{n+1}x^{2(n+1)} + nx^4\right]$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{F}_n,x,y)_{x=1} = 4n\left[\frac{\sqrt{n}}{n+1} + 1\right]$$

It follows from Table 1, $GA_e^v(\mathcal{F}_n) = 4n\left[\frac{\sqrt{n}}{n+1} + 1\right]$.

4. From Theorem 3.4,

$$M_e^v(\mathcal{S}_n,x,y) = nxy^2[2(xy)^2 + x^2 + 1]$$

$$D_y^{\frac{1}{2}}M_e^v(\mathcal{S}_n,x,y) = 4nx^3y^4 + n\sqrt{2}x^3y^2 + n\sqrt{2}xy^2$$

$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{S}_n,x,y)=4n\sqrt{3}x^3y^4+n\sqrt{6}x^3y^2+n\sqrt{2}xy^2$$

$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{S}_n,x,y)=4n\sqrt{3}x^7+n\sqrt{6}x^5+n\sqrt{2}x^3$$
$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{S}_n,x,y)=2\left[\frac{4n\sqrt{3}}{7}x^7+\frac{n\sqrt{6}}{5}x^5+\frac{n\sqrt{2}}{3}x^3\right]$$
$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{S}_n,x,y)_{x=1}=2n\left[\frac{4\sqrt{3}}{7}+\frac{\sqrt{6}}{5}+\frac{\sqrt{2}}{3}\right]$$

It follows from Table1, $GA_e^v(\mathcal{S}_n)=2n\left[\frac{4\sqrt{3}}{7}+\frac{\sqrt{6}}{5}+\frac{\sqrt{2}}{3}\right]$.

5. From Theorem 3.5,

$$M_e^v(\mathcal{H}_n,x,y)=nxy^3[(xy)^{n-1}+x^3y^{n-1}+2(xy)^3+x^3+1]$$
$$D_y^{\frac{1}{2}}M_e^v(\mathcal{H}_n,x,y)=n\sqrt{n+2}x^ny^{n+2}+n\sqrt{n+2}x^4y^{n+2}+2n\sqrt{6}x^{4y^6}+n\sqrt{3}x^4y^3+n\sqrt{3}xy^3$$
$$D_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{H}_n,x,y)=n\sqrt{n(n+2)}x^ny^{n+2}+2n\sqrt{n+2}x^4y^{n+2}+4n\sqrt{6}x^4y^6+2n\sqrt{3}x^4y^3$$
$$+n\sqrt{3}xy^3$$
$$JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{H}_n,x,y)=n\sqrt{n(n+2)}x^{2(n+1)}+2n\sqrt{n+2}x^{n+6}+4n\sqrt{6}x^{10}+2n\sqrt{3}x^7+n\sqrt{3}x^4$$
$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{H}_n,x,y)=2\left[\frac{n\sqrt{n(n+2)}}{2(n+1)}x^{2(n+1)}+\frac{2n(n+2)}{n+6}x^{n+6}+\frac{2n\sqrt{6}}{5}x^{10}+\frac{2n\sqrt{3}}{7}x^7\right.$$
$$\left.+\frac{n\sqrt{3}}{4}x^4\right]$$

$$2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}}M_e^v(\mathcal{H}_n,x,y)_{x=1}=2n\left[\frac{\sqrt{n(n+2)}}{2(n+1)}+\frac{2(n+2)}{n+6}+\frac{2\sqrt{6}}{5}+\frac{15\sqrt{3}}{28}\right]$$

It follows from Table 1, $GA_e^v(\mathcal{H}_n)=2n\left[\frac{\sqrt{n(n+2)}}{2(n+1)}+\frac{2(n+2)}{n+6}+\frac{2\sqrt{6}}{5}+\frac{15\sqrt{3}}{28}\right]$.

6. From Theorem 3.6,

$$M_e^v(\mathcal{H}_n^*,x,y)=n(xy)^2[x^{n-2}y^n+x^2y^n+2x^2y^4+(xy)^2+y^2+2(n-2)]$$

$$D_y^{\frac{1}{2}}M_e^v(\mathcal{H}_n^*,x,y)=n\sqrt{n+2}x^ny^{n+2}+n\sqrt{n+2}x^4y^{n+2}+2n\sqrt{6}x^4y^6+2n(xy)^4+$$

$$2nx^2y^4+2n(n-2)\sqrt{2}(xy)^2+nx^2y+nxy$$

$$
\begin{aligned}
D_x^{\frac{1}{2}} D_y^{\frac{1}{2}} M_e^v(\mathcal{H}_n^*, x, y) &= n\sqrt{n(n+2)}x^n y^{n+2} + 2n\sqrt{n+2}x^4 y^{n+2} + 4n\sqrt{6}x^4 y^6 + 4n(xy)^4 \\
&\quad +2n\sqrt{2}x^2 y^4 + 4n(n-2)(xy)^2 + n\sqrt{2}x^2 y + nxy \\
JD_x^{\frac{1}{2}} D_y^{\frac{1}{2}} M_e^v(\mathcal{H}_n^*, x, y) &= n\sqrt{n(n+2)}x^{2(n+1)} + 2n\sqrt{n+2}x^{n+6} + 4n\sqrt{6}x^{10} + 4nx^8 + 2n\sqrt{2}x^6 \\
&\quad +4n(n-2)x^4 + n\sqrt{2}x^3 + nx^2 \\
2S_x JD_x^{\frac{1}{2}} D_y^{\frac{1}{2}} M_e^v(\mathcal{H}_n^*, x, y) &= 2\left[\frac{n\sqrt{n(n+2)}}{2(n+1)}x^{2(n+1)} + \frac{2n\sqrt{n+2}}{n+6}x^{n+6} + \frac{2n\sqrt{6}}{5}x^{10} + \frac{n}{2}x^8\right. \\
&\quad \left. + \frac{n\sqrt{2}}{3}x^6 + n(n-2)x^4 + \frac{n\sqrt{2}}{3}x^3 + \frac{n}{2}x^2\right] \\
2S_x JD_x^{\frac{1}{2}} D_y^{\frac{1}{2}} M_e^v(\mathcal{H}_n^*, x, y)_{x=1} &= 2n\left[\frac{\sqrt{n(n+2)}}{2(n+1)} + \frac{2\sqrt{n+2}}{n+6} + \frac{2\sqrt{6}}{5} + \frac{2\sqrt{2}}{3} + n - 1\right]
\end{aligned}
$$

It follows from Table1, $GA_e^v(\mathcal{H}_n^*) = 2n\left[\frac{\sqrt{n(n+2)}}{2(n+1)} + \frac{2\sqrt{n+2}}{n+6} + \frac{2\sqrt{6}}{5} + \frac{2\sqrt{2}}{3} + n - 1\right]$.

1.2 Corollary

1. $N_e^v(\hat{S}_n) = (n-3)[\sqrt{2n-1} + \sqrt{n+3}] + 2[\sqrt{2(n-1)} + \sqrt{n+1} + (n-4)\sqrt{7} + \sqrt{6} + \sqrt{5}]$
2. $N_e^v(I_n) = (n-1)[\sqrt{2n-1} + \sqrt{n+3}] + 2[(n-2)\sqrt{7} + \sqrt{6} + \sqrt{5}]$
3. $N_e^v(\mathcal{F}_n) = 2n[\sqrt{2(n+1)} + 2\sqrt{n} + 2]$
4. $N_e^v(\mathcal{S}_n) = n[2\sqrt{7} + \sqrt{5} + \sqrt{3}]$
5. $N_e^v(\mathcal{H}_n) = n[\sqrt{2(n+1)} + \sqrt{n+6} + 2\sqrt{10} + \sqrt{7} + 2]$
6. $N_e^v(\mathcal{H}_n^*) = n[\sqrt{2(n+1)} + \sqrt{n+6} + 2\sqrt{10} + \sqrt{6} + \sqrt{3} + 3\sqrt{2} + 4(n-2)]$

Proof:

1. From Theorem 3.1,

$$
\begin{aligned}
M_e^v(\hat{\mathrm{S}}_n, x, y) &= x^{n-1}[2y^{n-1} + (n-3)y^n] + 2x^2(y^3 + y^{n-1}) + 2x^3(y^3 + y^4 + y^n) + (n-5)x^3[2y^4 + y^n] \\
JM_e^v(\hat{\mathrm{S}}_n, x, y) &= 2x^{2(n-1)} + (n-3)x^{2n-1} + 2(n-4)x^7 + 2x^{n+1} + 2x^6 + (n-3)x^{n+3} + 2x^5 \\
D_x^{\frac{1}{2}} JM_e^v(\hat{\mathrm{S}}_n, x, y) &= \sqrt{2(n-1)}2x^{2(n-1)} + \sqrt{2n-1}(n-3)x^{2n-1} + \sqrt{7}2(n-4)x^7 + \sqrt{n+1}2x^{n+1} \\
&\quad +\sqrt{6}2x^6 + \sqrt{n+3}(n-3)x^{n+3} + \sqrt{5}2x^5 \\
D_x^{\frac{1}{2}} JM_e^v(\hat{\mathrm{S}}_n, x, y)\Big|_{x=1} &= (n-3)[\sqrt{2n-1} + \sqrt{n+3}] + 2[\sqrt{2(n-1)} + \sqrt{n+1} + (n-4)\sqrt{7} + \sqrt{6} + \sqrt{5}]
\end{aligned}
$$

It follows from Table 1,

$N_e^v(\hat{S}_n) = (n-3)[\sqrt{2n-1} + \sqrt{n+3}] + 2[\sqrt{2(n-1)} + \sqrt{n+1} + (n-4)\sqrt{7} + \sqrt{6} + \sqrt{5}]$.

2. From Theorem 3.2, $M_e^v(I_n, x, y) = (n-1)x^{n-1}y^n + 2x^3(y^3 + y^4 + y^n) + (n-3)x^3(2y^4 + y^n) + 2x^2y^3$

$$JM_e^v(I_n, x, y) = (n-1)x^{2n-1} + (n-1)x^{n+3} + 2(n-2)x^7 + 2x^6 + 2x^5$$

$$D_x^{\frac{1}{2}}JM_e^v(I_n, x, y) = (n-1)\sqrt{2n-1}x^{2n-1} + (n-1)\sqrt{n+3}x^{n+3} + 2(n-2)\sqrt{7}x^7 + 2\sqrt{6}x^6 + 2\sqrt{5}x^5$$

$$\left. D_x^{\frac{1}{2}}JM_e^v(I_n, x, y)\right|_{x=1} = (n-1)[\sqrt{2n-1} + \sqrt{n+3}] + 2[(n-2)\sqrt{7} + \sqrt{6} + \sqrt{5}]$$

It follows from Table 1, $N_e^v(I_n) = (n-1)[\sqrt{2n-1} + \sqrt{n+3}] + 2[(n-2)\sqrt{7} + \sqrt{6} + \sqrt{5}]$.

3. From Theorem 3.3, $M_e^v(\mathcal{F}_n, x, y) = 2n(xy)^{2n} + 2n(xy)^2 + 2nx^2y^{2n}$

$$JM_e^v(\mathcal{F}_n, x, y) = 2nx^{4n} + 2nx^4 + 2nx^{2(n+1)}$$

$$D_x^{\frac{1}{2}}JM_e^v(\mathcal{F}_n, x, y) = 4n\sqrt{n}x^{4n} + 4nx^4 + 2n\sqrt{2(n+1)}x^{2(n+1)}$$

$$\left. D_x^{\frac{1}{2}}JM_e^v(\mathcal{F}_n, x, y)\right|_{x=1} = 2n[\sqrt{2(n+1)} + 2\sqrt{n} + 2]$$

It follows from Table 1, $N_e^v(\mathcal{F}_n) = 2n[\sqrt{2(n+1)} + 2\sqrt{n} + 2]$.

4. From Theorem 3.4, $M_e^v(\mathcal{S}_n, x, y) = nxy^2[2(xy)^2 + x^2 + 1]$

$$JM_e^v(\mathcal{S}_n, x, y) = 2nx^7 + nx^5 + nx^3$$

$$D_x^{\frac{1}{2}}JM_e^v(\mathcal{S}_n, x, y) = 2n\sqrt{7}x^7 + n\sqrt{5}x^5 + n\sqrt{3}x^3$$

$$\left. D_x^{\frac{1}{2}}JM_e^v(\mathcal{S}_n, x, y)\right|_{x=1} = n[2\sqrt{7} + \sqrt{5} + \sqrt{3}]$$

It follows from Table 1, $N_e^v(\mathcal{S}_n) = n[2\sqrt{7} + \sqrt{5} + \sqrt{3}]$.

5. From Theorem 3.5, $M_e^v(\mathcal{H}_n, x, y) = nxy^3[(xy)^{n-1} + x^3y^{n-1} + 2(xy)^3 + x^3 + 1]$

$$JM_e^v(\mathcal{H}_n, x, y) = nx^{2(n+1)} + nx^{n+6} + 2nx^{10} + nx^7 + nx^4$$

$$D_x^{\frac{1}{2}}JM_e^v(\mathcal{H}_n, x, y) = n\sqrt{2(n+1)}x^{2(n+1)} + n\sqrt{n+6}x^{n+6} + 2n\sqrt{10}x^{10} + n\sqrt{7}x^7 + 2nx^4$$

$$\left. D_x^{\frac{1}{2}}JM_e^v(\mathcal{H}_n, x, y)\right|_{x=1} = n[\sqrt{2(n+1)} + \sqrt{n+6} + 2\sqrt{10} + \sqrt{7} + 2]$$

It follows from Table 1, $N_e^v(\mathcal{H}_n) = n[\sqrt{2(n+1)} + \sqrt{n+6} + 2\sqrt{10} + \sqrt{7} + 2]$.

6. From Theorem 3.6, $M_e^v(\mathcal{H}_n^*, x, y) = n(xy)^2[x^{n-2}y^n + x^2y^n + 2x^2y^4 + (xy)^2 + y^2 + 2(n-2)] + nx^2y + nxy$

$$
\begin{aligned}
JM_e^v(\mathcal{H}_n^*, x, y) &= nx^{2(n+1)} + nx^{n+6} + 2nx^{10} + nx^8 + nx^6 + 2n(n-2)x^4 + nx^3 + nx^2 \\
D_x^{\frac{1}{2}} JM_e^v(\mathcal{H}_n^*, x, y) &= n\sqrt{2(n+1)}x^{2(n+1)} + n\sqrt{n+6}x^{n+6} + 2n\sqrt{10}x^{10} + n\sqrt{8}x^8 + n\sqrt{6}x^6 \\
&\quad +4n(n-2)x^4 + n\sqrt{3}x^3 + n\sqrt{2}x^2 \\
D_x^{\frac{1}{2}} JM_e^v(\mathcal{H}_n^*, x, y)\Big|_{x=1} &= n[\sqrt{2(n+1)} + \sqrt{n+6} + 2\sqrt{10} + \sqrt{6} + \sqrt{3} + 3\sqrt{2} + 4(n-2)]
\end{aligned}
$$

It follows from Table 1, $N_e^v(\mathcal{H}_n^*) = n[\sqrt{2(n+1)} + \sqrt{n+6} + 2\sqrt{10} + \sqrt{6} + \sqrt{3} + 3\sqrt{2} + 4(n-2)]$.

5. Conclusion:

In conclusion, this study introduced a new version of *M*-polynomial that was known by M_e^v-polynomial with some related *ve*-topological indices. The obtained results extend existing results in graph theory and provide a symmetric framework for analyzing similar graph families. Our findings highlight the structural properties of the presented special graphs and illustrate the importance of M_e^v-polynomial in simplifying the computation of various *ve*-indices, especially GA_e^v and N_e^v. Despite of the limitation to specific graphs, the methodology can be generalized to other graph structures such as chemical graphs, offering a straight way for future theoretical development. Future research might focus on calculating the proposed graph polynomial and indices for various graphs.

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متعددة الحدود – M_e^v والمؤشرالطبولوجي رأس – حافة لبعض بيانات

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قسم الرياضيات، كلية العلوم، جامعة السليمانية، السليمانية، إقليم كردستان العراق، العراق

المستخلص

نفرض ان G بيان بسيط يحتوي على n من الرؤوس و m من الحافات. ان متعددة الحدود M تعتمد على العلاقة بين الرؤوس والحافات تعرف بمتعددة الحدود من نوع الرأس الى الحافة و(متعددة – M_e^v) تعرف على النحو التالي: $M_e^v(G,x,y)=\sum_{i=1}^{n}\sum_{j=1}^{d_G(v_i)} x^{d_G(v_i)}y^{d_G(e_j)}$ حيث $d_G(v_i)$ درجة الرأس v_i و $d_G(e_j)$ درجة الحافة e_j والتي تُعرّف على أنها عدد المتجاورات المتصلات بالحافة e_j. في هذا البحث تم حساب متعددة الحدود-M_e^v ومؤشرين طوبولوجيين جديدين مقترحين من نوع الرأس-حافة GA_e^v , و N_e^v لبعض البيانات الخاصة.

الكلمات المفتاحية : كثيرة حدود المخطط، كثيرة حدود -*M_e^v* ، دليل **ve** الطوبولوجي.