M-polynomial and CoM-polynomial of Hat-graphs

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ABSTRACT

The M-Polynomial and CoM-polynomial are considered as two of the most important degree-based graph polynomials, since from them we can obtain almost all degree based topological indices and coindices respectively. In this paper these two polynomials will be computed for some new defined graphs from the basic well-known graphs, such graphs will be known as hat-graphs. Hat-graphs have been constructed by adding new vertices and edges for the basic graphs in a special way. The special graphs that have been the base of these new graphs are path, cycle, complete, star, wheel, and complete bipartite graph.

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1. Introduction

A simple connected graph G consists of two sets; $V(G)$ is the set of vertices, and $E(G)$ is the set of unordered pairs of elements of $V(G)$ called edges [1.-3]. The number of vertices and edges of G determine it's order and size, which are represented by n, m respectively. Two vertices are called adjacent if the have an edge between them, by degree of a vertex v we mean the number of all edges incident to v, which is denoted by $d_G(v)$. The complement of a graph G is a new graph $\bar{G}$ having the vertex set $V(G)$, two vertices are adjacent in $\bar{G}$ iff they are not adjacent in G, and the set of all edges of G is denoted by $\overline{E(G)}[1-3]$. By an i-vertex we mean a vertex of degree i, n_i is the number of i-vertices and (i,j)-edge refers to an edge joining an i-vertex to a j-vertex [4, 5]. A graph in which every pair of vertices are adjacent is called Complete graph and denoted by $K_n[2]$. A graph in which every vertex has a degree $r, (r \in N)$ is called r-regular [3]. A connected graph with order n which is 2 -regular is called Cycle and denoted by C_n [1]. Path graph P_n is obtained from C_n by removing an edge, while the graph obtained from C_n by adding a new vertex and joining all n vertices of C_n to the new vertex is called Wheel and denoted by W_n [1]. A complete bipartite graph $K_{n,m}$ with order $n+m$ is a graph that vertex set can be partition in two two sets V_1 and V_2 such that, no two vertices in the same partition are adjacent and every vertex of V_1 is adjacent to every vertices of V_2. In particular, the Star graph S_n is the complete bipartite graph $K_{1,n}$ [1,2]. A graph invariant with polynomial values are graph polynomial [6]. Many polynomials have been introduced such as chromatic polynomial whose values give the number of coloring of the graph properly with a given number of colors [7], Hosoya polynomial based on distances [8, 9], characteristic polynomial based on the matrix of graphs [10], M-polynomial and CoM-polynomial based on degrees [11, 12], etc. In this research we focus on M-polynomial, CoM-polynomial and compute them for certain graphs. The M-Polynomial is an important degree based graph polynomial which was defined by Deutsch and Klavžar in 2015 [11]. For a graph G, the M-Polynomial is defined as:

$$M(G,x,y) = \sum_{\delta(G) \le i \le j \le \Delta(G)} m_{i,j} x^i y^j \qquad (1)$$

where $m_{i,j}$ is the number of (i,j)-edges and $\delta(G), \Delta(G)$ represent minimum and maximum degrees of vertices of G respectively [11]. There are many studies about the M-Polynomial including computation of M-Polynomial for Some composite graphs, book graph and some nanostructures graph in [13-15]. Additionally, Basavanagoud et al. calculated the M-Polynomial of some graph operations and cycle related graphs in [16]. In contrast to other graph polynomials this polynomial makes it simple to compute more than one topological index, including Atom bond connectivity index, Geometric connectivity index and more indices by using specific derivative, integral, or some times both. [11, 14-16] provide formulas for computing those indices from the M-Polynomial. In 2022 the concept of M-Polynomial extended for non-adjacent pair of vertices by Kirmani et al. They defined a new polynomial called CoM-Polynomial [12], since then some publications can be found on this polynomial such as [12, 17--20]. The CoM-Polynomial of a graph G is defined as:

$$\mathrm{CoM}(G,x,y) = \bar{M}(G,x,y) = \sum_{\delta(G)\leq i\leq j\leq\Delta(G)} \bar{m}_{i,j}x^i y^j \qquad (2)$$

where $\bar{m}_{i,j}$ is the number of all pairs of vertices (u,v) with $d_G(u)=i$, and $d_G(v)=j$ such that $uv \notin E(G)$ that is; $\bar{m}_{i,j} = |\{uv \notin E(G); d_G(u)=i \text{ and } d_G(v)=j\}|$ and $\delta(G), \Delta(G)$ are minimum and maximum degrees of vertices of G respectively [12, 17]. While computing $\bar{m}_{i,j}$ the following Lemma is helpful;

Lemma 1.1 [5] Let G be a connected graph of order n. Then

$$\bar{m}_{i,j} = \begin{cases} \dfrac{n_i(n_i-1)}{2} - m_{i,j} & \text{if } i=j \\ n_i n_j - m_{i,j} & \text{if } i<j \end{cases} \qquad (3)$$

2. Main Results

In this section some new special graphs will be constructed from the well-known basic graph by adding new vertices and edges in a special way, these graphs will be known as Hat-graphs. Moreover, the M-polynomial and Co-M-polynomial will be computed for each such graphs.

Definition 2.1 Let K_n be a Complete graph with vertex set $\{u_1,u_2,u_3,\dots,u_n\}$. We define a new graph HatComplete K_n^H by adding new vertices $\{v_1,v_2,v_3,\dots,v_{2n}\}$ and edges $\{u_i v_{2(i-1)}, u_i v_{2i-1}, v_{2i-1}v_{2i}; i=1,2,3,\dots,n \text{ and } v_0=v_{2n}\}$. The new graph K_n^H has $3n$ vertices and $\frac{n(n+5)}{2}$ edges with diameter 3 (see Figure 1).

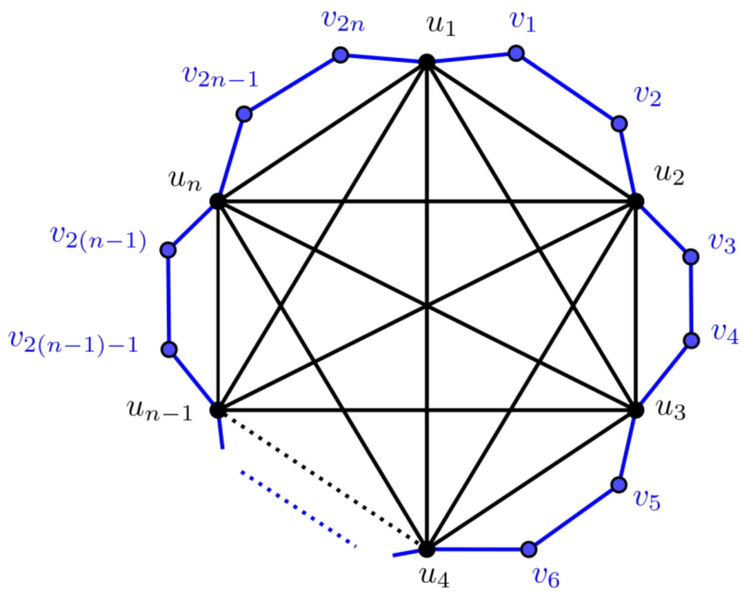

Figure 1: The Hat-Complete graph K_n^H

Theorem 2.1 The M-polynomial and CoM-polynomial of K_n^H are:

$$M(K_n^H, x, y) = n(xy)^2\left[\frac{n-1}{2}(xy)^{n-1} + 2y^{n-1} + 1\right] \tag{4}$$
$$\bar{M}(K_n^H, x, y) = 2n(n-1)(xy)^2(y^{n-1} + 1) \tag{5}$$

Proof: We see that $|V(K_n^H)| = 3n$ and $|E(K_n^H)| = \frac{n(n+5)}{2}$. On the other hand, $\left|E\left(\overline{K_n^H}\right)\right| = \binom{3n}{2} - |E(K_n^H)| = 4n(n-1)$. Vertices of K_n^H have degrees 2, and $n+1$, so based on degree $V(K_n^H)$ can be classified into class: $n_2 = 2n$, and $n_{n+1} = n$. Then by using the above information and Lemma 1.1 we have the following table:

Table 1: Edge partitions and number of edges of K_n^H and $\overline{K_n^H}$.

$(i,j)=(d_G(u),d_G(v))$	$m_{i,j}$	$\bar{m}_{i,j}$
$(2,2)$	n	$2n(n-1)$
$(2,n+1)$	$2n$	$2n(n-1)$
$(n+1,n+1)$	$\binom{n}{2}$	0
Σ	$\frac{n(n+5)}{2}$	$4n(n-1)$

Then,

$$\begin{aligned} M(K_n^H,x,y) &= \sum_{\delta(K_n^H)\le i\le j\le\Delta(K_n^H)} m_{i,j}x^i y^j \\ &= n(xy)^2+2nx^2y^{n+1}+\binom{n}{2}(xy)^{n+1} \\ &= n(xy)^2+2nx^2y^{n+1}+\frac{n(n-1)}{2}(xy)^{n+1} \\ &= n(xy)^2\left[\frac{n-1}{2}(xy)^{n-1}+2y^{n-1}+1\right]. \end{aligned}$$

And,

$$\begin{aligned} \bar{M}(K_n^H,x,y) &= \sum_{\delta(K_n^H)\le i\le j\le\Delta(K_n^H)} \bar{m}_{i,j}x^i y^j \\ &= 2n(n-1)(xy)^2+2n(n-1)x^2y^{n+1}+0 \\ &= 2n(n-1)(xy)^2(y^{n-1}+1). \end{aligned}$$

Definition 2.2 Let P_n be a Path with vertex set $\{u_1,u_2,u_3,\dots,u_n\}$. We define a new graph Hat-Path P_n^H by adding new vertices $\{v_1,v_2,v_3,\dots,v_{2(n-1)}\}$ and edges $\{u_iv_{2(i-1)},u_iv_{2i-1};i=2,3,\dots,n-1\text{ and }\}\cup\{u_1v_1,u_nv_{2(n-1)}\}$. The new graph P_n^H has $3n-2$ vertices and $4(n-1)$ edges with diameter $n+1$ (see Figure 2).

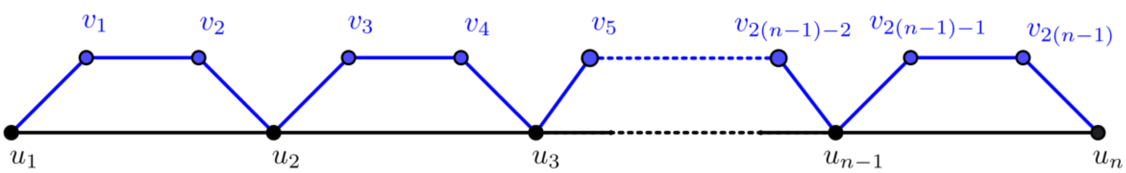

Figure 2: The Hat-Path graph P_n^H

Theorem 2.2 The M-polynomial and CoM-polynomial of P_n^H are:

$$M(P_n^H, x, y) = (xy)^2[((n-3)x^2 + 2(n-1))y^2 + n + 1] \tag{6}$$

$$\bar{M}(P_n^H, x, y) = (xy)^2\left[\frac{n^2-7n+12}{2}(xy)^2 + (2n^2-6n+2)y^2 + 2n^2 - 2n - 1\right] \tag{7}$$

Proof: We see that $|V(P_n^H)| = 3n-2$ and $|E(P_n^H)| = 4(n-1)$. On the other hand, $\left|E\left(\overline{P_n^H}\right)\right| = \binom{3n-2}{2} - |E(P_n^H)| = \frac{9n^2-23n+14}{2}$. Vertices of P_n^H have degrees 2, and 4, so based on degree $V(P_n^H)$ can be classified into class: $n_2 = 2n$, and $n_4 = n-2$. Then by using the above information and Lemma 1.1 we have the following table:

Table 2: Edge partitions and number of edges of P_n^H and $\overline{P_n^H}$.

$(i,j) = (d_G(u), d_G(v))$	$m_{i,j}$	$\bar{m}_{i,j}$
(2,2)	$n+1$	$2n^2-2n-1$
(2,4)	$2(n-1)$	$2n(n-3)+2$
(4,4)	$n-3$	$\binom{n-3}{2}$
Σ	$4(n-1)$	$\frac{9n^2-23n+14}{2}$

Then,

$$\begin{aligned} M(P_n^H, x, y) &= \sum_{\delta(P_n^H) \le i \le j \le \Delta(P_n^H)} m_{i,j} x^i y^j \\ &= (n+1)(xy)^2 + 2(n-1)x^2y^4 + (n-3)(xy)^4 \\ &= (xy)^2[((n-3)x^2 + 2(n-1))y^2 + n + 1]. \end{aligned}$$

And,

$$\begin{aligned} \bar{M}(P_n^H, x, y) &= \sum_{\delta(P_n^H) \le i \le j \le \Delta(P_n^H)} \bar{m}_{i,j} x^i y^j \\ &= (2n^2 - 2n - 1)(xy)^2 + [2n(n-3) + 2]x^2y^4 + \binom{n-3}{2}(xy)^4 \\ &= (xy)^2\left[\frac{n^2 - 7n + 12}{2}(xy)^2 + (2n^2 - 6n + 2)y^2 + 2n^2 - 2n - 1\right]. \end{aligned}$$

Definition 2.3 Let C_n be a Cycle with vertex set $\{u_1, u_2, u_3, \dots, u_n\}$. We define a new graph Hat-Cycle C_n^H by adding new vertices $\{v_1, v_2, v_3, \dots, v_{2n}\}$ and edges $\{u_i v_{2(i-1)}, u_i v_{2i-1}, v_{2i-1} v_{2i};$ $i = 1,2,3, \dots, n$ and $v_0 = v_{2n}\}$. The new graph C_n^H has $3n$ vertices and $4n$ edges with diameters $\frac{n+4}{2}$ when n is an even integer and $\frac{n+3}{2}$ when n is an odd integer (see Figure 3).

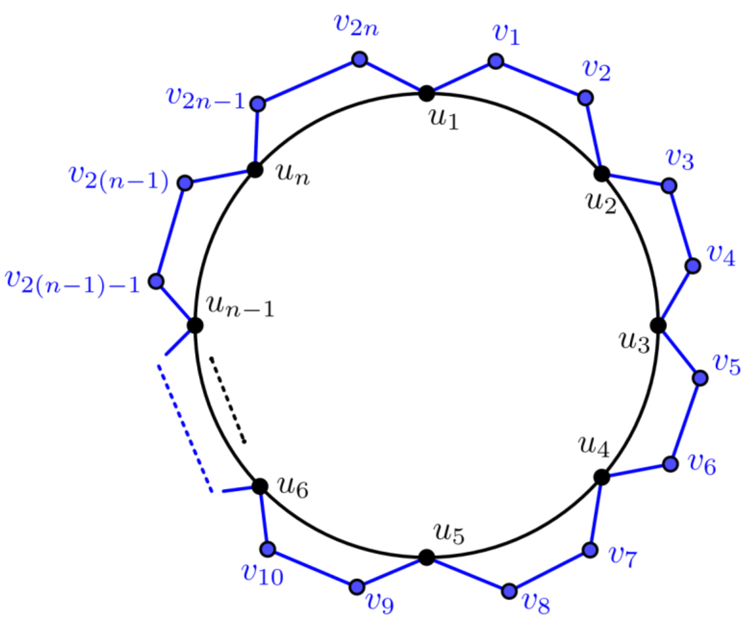

Figure 3: The Hat-Cycle graph C_n^H

Theorem 2.3 The M-polynomial and CoM-polynomial of C_n^H are:

$$M(C_n^H, x, y) = n(xy)^2[(xy)^2 + 2y^2 + 1] \quad (8)$$

$$\bar{M}(C_n^H, x, y) = 2n(xy)^2\left[\frac{n-3}{4}(xy)^2 + (n-1)y^2 + n - 1\right] \quad (9)$$

Proof: We see that $|V(C_n^H)| = 3n$ and $|E(C_n^H)| = 4n$. On the other hand, $\left|E\left(\overline{C_n^H}\right)\right| = \binom{3n}{2} - |E(C_n^H)| = \frac{n(9n-11)}{2}$. Vertices of C_n^H have degrees 2 , and 4 , so based on degree $V(C_n^H)$ can be classified into class: $n_2 = 2n$, and $n_4 = n$. Then by using the above information and Lemma 1.1 we have the following table:

Table 3: Edge partitions and number of edges of C_n^H and $\overline{C_n^H}$.

$(i,j) = (d_G(u), d_G(v))$	$m_{i,j}$	$\bar{m}_{i,j}$
(2,2)	n	$2n(n-1)$
(2,4)	$2n$	$2n(n-1)$
(4,4)	n	$\frac{n(n-3)}{2}$
Σ	$4n$	$\frac{n(9n-11)}{2}$

Then,

$$\begin{aligned} M(C_n^H, x, y) &= \sum_{\delta(C_n^H) \le i \le j \le \Delta(C_n^H)} m_{i,j} x^i y^j \\ &= n(xy)^2 + 2nx^2y^4 + n(xy)^4 \\ &= n(xy)^2[(xy)^2 + 2y^2 + 1]. \end{aligned}$$

And,

$$\begin{aligned}\bar{M}(C_n^H,x,y) &= \sum_{\delta(C_n^H)\le i\le j\le \Delta(C_n^H)} \bar{m}_{i,j}x^iy^j \\ &= 2n(n-1)(xy)^2+2n(n-1)x^2y^4+\frac{n(n-3)}{2}(xy)^4 \\ &= 2n(xy)^2\left[\frac{n-3}{4}(xy)^2+(n-1)y^2+n-1\right].\end{aligned}$$

Definition 2.4 Let S_n be a Star with vertex set $\{u_1,u_2,u_3,\dots,u_n,u_{n+1}\}$. We define a new graph Hat-Star S_n^H by adding new vertices $\{v_1,v_2,v_3,\dots,v_{2n}\}$ and edges $\{u_iv_{2(i-1)},u_iv_{2i-1},v_{2i-1}v_{2i};i=1,2,3,\dots,n \text{ and } v_0=v_{2n}\}$. The new graph S_n^H has $3n+1$ vertices and $4n$ edges with diameter 4 (see Figure4).

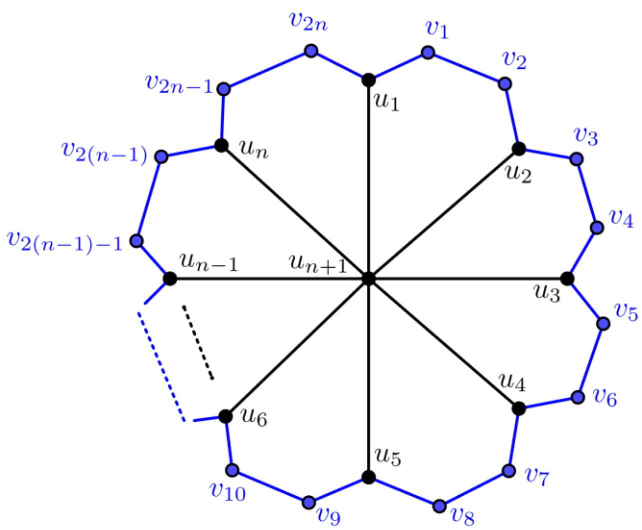

Figure 4: The Hat-Star graph S_n^H

Theorem 2.4 The M-polynomial and CoM-polynomial of S_n^H are:

$$M(S_n^H,x,y)=n(xy)^2[xy^{n-2}+2y+1] \tag{10}$$

$$\bar{M}(S_n^H,x,y)=2n(n-1)(xy)^2\left[\frac{y^{n-2}}{n-1}+\frac{xy}{4}+y+1\right] \tag{11}$$

Proof: We see that $|V(S_n^H)|=3n+1$ and $|E(S_n^H)|=4n$. On the other hand, $\left|E\left(\overline{S_n^H}\right)\right|=\binom{3n+1}{2}-|E(S_n^H)|=\frac{n(9n-5)}{2}$. Vertices of S_n^H have degrees 2,3 , and n, so based on degree $V(S_n^H)$

can be classified into class: $n_2 = 2n, n_3 = n$ and $n_n = 1$. Then by using the above information and Lemma 1.1 we have the following table:

Table 4: Edge partitions and number of edges of S_n^H and $\overline{S_n^H}$.

$(i,j) = (d_G(u), d_G(v))$	$m_{i,j}$	$\bar{m}_{i,j}$
(2,2)	n	$2n(n-1)$
(2,3)	$2n$	$2n(n-1)$
$(2,n)$	0	$2n$
(3,3)	0	$\binom{n}{2}$
$(3,n)$	n	0
(n,n)	0	0
$\Sigma\Sigma$	$4n$	$\frac{n(9n-5)}{2}$

Then,

$$M(S_n^H, x, y) = \sum_{\delta(S_n^H)\le i\le j\le \Delta(S_n^H)} m_{i,j} x^i y^j$$
$$= n(xy)^2 + 2nx^2y^3 + nx^3y^n$$
$$= n(xy)^2[xy^{n-2} + 2y + 1].$$

And,

$$\bar{M}(S_n^H, x, y) = \sum_{\delta(S_n^H)\le i\le j\le \Delta(S_n^H)} \bar{m}_{i,j} x^i y^j$$
$$= 2n(n-1)(xy)^2 + 2n(n-1)x^2y^3 + 2nx^2y^n + \binom{n}{2}(xy)^3$$
$$= 2n(n-1)(xy)^2\left[\frac{y^{n-2}}{n-1} + \frac{xy}{4} + y + 1\right].$$

Definition 2.5 Let W_n be a Wheel with vertex set $\{u_1, u_2, u_3, \ldots, u_n, u_{n+1}\}$. We define a new graph Hat-Wheel W_n^H by adding new vertices $\{v_1, v_2, v_3, \ldots, v_{2n}\}$ and edges $\{u_i v_{2(i-1)}, u_i v_{2i-1}, v_{2i-1} v_{2i}; i = 1,2,3, \ldots, n \text{ and } v_0 = v_{2n}\}$. The new graph W_n^H has $3n + 1$ vertices and $5n$ edges with diameter 4 (see Figure 5).

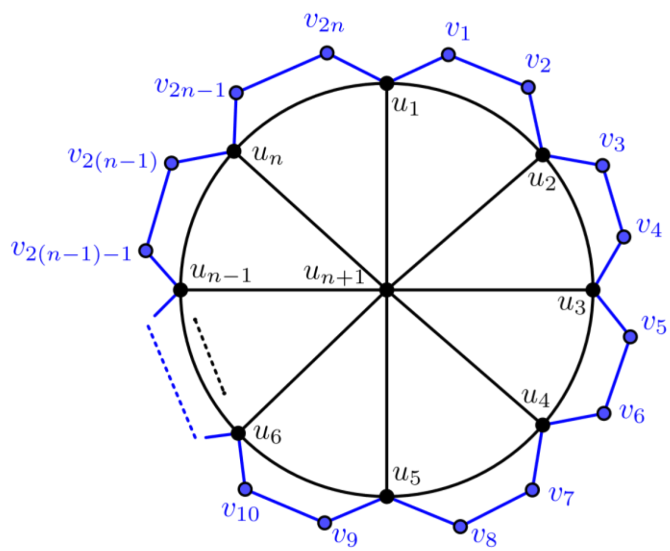

Figure 5: The Hat-Wheel graph W_n^H

Theorem 2.5 The M-polynomial and CoM-polynomial of W_n^H are:

$$M(W_n^H, x, y) = n(xy)^2[x^3y^{n-2} + (xy)^3 + 2y^3 + 1] \tag{12}$$

$$\bar{M}(W_n^H, x, y) = 2n(xy)^2\left[y^{n-2} + \frac{n-3}{4}(xy)^3 + (n-1)y^3 + n - 1\right] \tag{13}$$

Proof: We see that $|V(W_n^H)| = 3n + 1$ and $|E(W_n^H)| = 5n$. On the other hand, $\left|E\left(\overline{W_n^H}\right)\right| = \binom{3n+1}{2} - |E(W_n^H)| = \frac{n}{2}[9n - 7]$. Vertices of W_n^H have degrees 2,5 , and n, so based on degree $V(W_n^H)$ can be classified into class: $n_2 = 2n, n_5 = n$ and $n_n = 1$. Then by using the above information and Lemma 1.1 we have the following table:

Table 5: Edge partitions and number of edges of W_n^H and $\overline{W_n^H}$.

$(i,j) = (d_G(u), d_G(v))$	$m_{i,j}$	$\bar{m}_{i,j}$
$(2,2)$	n	$2n(n-1)$
$(2,5)$	$2n$	$2n(n-1)$
$(2,n)$	0	$2n$
$(5,5)$	n	$\frac{n(n-3)}{2}$
$(5,n)$	n	0
(n,n)	0	0
$\sum$	$5n$	$\frac{n}{2}[9n-7]$

Then,

$$\begin{aligned} M(W_n^H, x, y) &= \sum_{\delta(W_n^H)\le i\le j\le\Delta(W_n^H)} m_{i,j}x^i y^j \\ &= n(xy)^2 + 2nx^2y^5 + n(xy)^5 + nx^5y^n \\ &= n(xy)^2[x^3y^{n-2} + (xy)^3 + 2y^3 + 1]. \end{aligned}$$

And,

$$\begin{aligned} \bar{M}(W_n^H, x, y) &= \sum_{\delta(W_n^H)\le i\le j\le\Delta(W_n^H)} \bar{m}_{i,j}x^i y^j \\ &= 2n(n-1)(xy)^2 + 2n(n-1)x^2y^5 + 2nx^2y^n + \frac{n(n-3)}{2}(xy)^5 \\ &= 2n(xy)^2\left[y^{n-2} + \frac{n-3}{4}(xy)^3 + (n-1)y^3 + n - 1\right]. \end{aligned}$$

Definition 2.6 Let $K_{n,m}$ be a Complete bipartite graph ($n,m \geq 2$) with vertex set $\{u_1,u_2,u_3,\dots,u_n,u'_1,u'_2,u'_3,\dots,u'_m\}$. We define a new graph Hat-Complete bipartite $K^H_{n,m}$ by adding new vertices $\{v_1,v_2,v_3,\dots,v_{2(n+m)}\}$ and edges $\{u_iv_{2i-1},u_iv_{2i};i=1,2,3,\dots,n\}\cup\{u'_jv_{2(n+j)-1},u'_jv_{2(n+j)};j=1,2,\dots,m\}$ $\cup\{v_{2i}v_{2i+1};i=1,2,\dots,n+m$ and $v_{2(n+m)+1=v_1}\}$. The new graph $K^H_{n,m}$ has $3(n+m)$ vertices and $3(n+m)+nm$ edges with diameter 4 (see Figure 6).

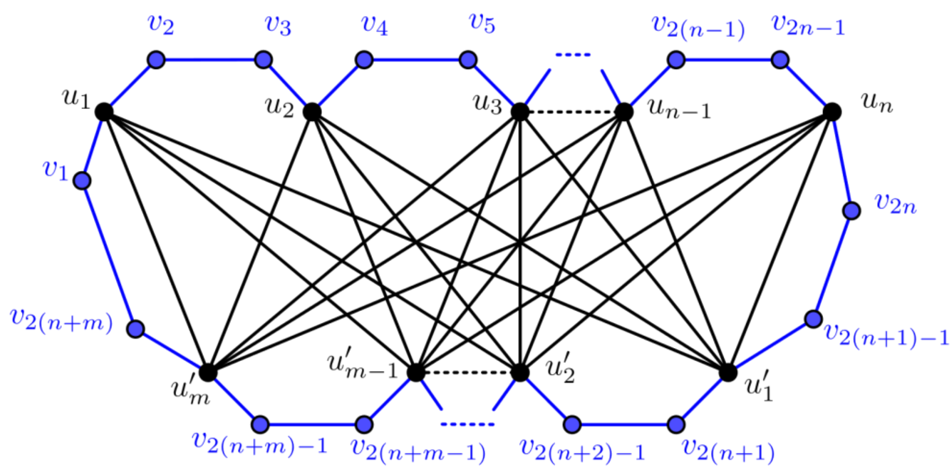

Figure 6: The Hat-Complete bipartite graph $K^H_{n,m}$

Theorem 2.6 The M-polynomial and CoM-polynomial of $K^H_{n,m}$ are:

$$M\left(K^H_{n,m},x,y\right)=(xy)^2[nmx^ny^m+2my^n+2ny^m+n+m] \tag{14}$$

$$\bar{M}\left(K^H_{n,m},x,y\right)=(xy)^2\left[2(n+m-1)(ny^m+my^n+(n+m))+\binom{n}{2}(xy)^m+\binom{m}{2}x^ny^m\right] \tag{15}$$

Proof: We see that $\left|V\left(K^H_{n,m}\right)\right|=3(n+m)$ and $\left|E\left(K^H_{n,m}\right)\right|=3(n+m)+nm$. On the other hand, $\left|E\left(\overline{K^H_{n,m}}\right)\right|=\binom{3(n+m)}{2}-\left|E\left(K^H_{n,m}\right)\right|=\frac{1}{2}[9(n^2+m^2)-9(n+m)+16nm]$. Vertices of $K^H_{n,m}$ have degrees $2,n+2$, and $m+2$, so based on degree $V\left(K^H_{n,m}\right)$ can be classified into class: $n_2\stackrel{=}{=}2(n+m), n_{n+2}=m$ and $n_{m+2}=n$. Then by using the above information and Lemma 1.1 we have the following table:

Table 6: Edge partitions and number of edges of $K_{n,m}^{H}$ and $\overline{K_{n,m}^{H}}$.

$(i,j) = (d_G(u), d_G(v))$	$m_{i,j}$	$\bar{m}_{i,j}$
$(2,2)$	$n+m$	$2(n+m)(n+m-1)$
$(2,n+2)$	$2m$	$2m(n+m-1)$
$(2,m+2)$	$2n$	$2n(n+m-1)$
$(n+2,m+2)$	0	$\binom{m}{2}$
$(n+2,m+2)$	nm	0
$(m+2,m+2)$	0	$\binom{n}{2}$
Σ	$3(n+m)+nm$	$\frac{1}{2}[9(n^2+m^2)-9(n+m)+16nm]$

Then,

$$\begin{aligned} M(K_{n,m}^{H},x,y) &= \sum_{\delta(K_{n,m}^{H})\leq i\leq j\leq \Delta(K_{n,m}^{H})} m_{i,j}x^i y^j \\ &= (n+m)(xy)^2 + 2mx^2y^{n+2} + 2nx^2y^{m+2} + nmx^{n+2}y^{m+2} \\ &= (xy)^2[nmx^n y^m + 2my^n + 2ny^m + n + m] \end{aligned}$$

And,

$$\begin{aligned} \bar{M}(K_{n,m}^{H},x,y) &= \sum_{\delta(K_{n,m}^{H})\leq i\leq j\leq \Delta(K_{n,m}^{H})} \bar{m}_{i,j}x^i y^j \\ &= 2(n+m)(n+m-1)(xy)^2 + 2m(n+m-1)x^2y^{n+2} + 2n(n+m-1)x^2y^{m+2} \\ &+ \binom{m}{2}x^{n+2}y^{m+2} + \binom{n}{2}(xy)^{m+2} \\ &= (xy)^2\left[2(n+m-1)(ny^m + my^n + (n+m)) + \binom{n}{2}(xy)^m + \binom{m}{2}x^n y^m\right]. \end{aligned}$$

References

[1] N. Biggs, E.K. Lloyd, R.J. Wilson, Graph Theory 1736-1936, Oxford University Press, (1986), https://dl.acm.org/doi/abs/10.5555/39804

[2] C. Vasudev, Graph theory with applications. New Age International, (2006), https://books.google.iq/books?id=Nb4iycwAR2IC

[3] G. Chartrand, P. Zhang, Chromatic graph theory, Chapman and Hall/CRC, (2008), https://doi.org/10.1201/9781584888017

[4] S. Alikhani, R. Hasni, N.E. Arif, On the atom-bond connectivity index of some families of dendrimers, J. Comp. Th. Nanoscience, 11 (2014) 1802-1805, https://doi.org/10.1166/jctn.2014.3570

[5] M. Berhe, C. Wang, Computation of certain topological coindices of graphene sheet and $C_4C_8(S)$ nanotubes and nanotorus, Appl. Math. Nonlinear Sci., 4(2019)455-468, https://doi.org/10.2478/AMNS.2019.2.00043

[6] M. Trinks, Graph polynomials and their representations, PhD thesis. Semantic scholar, (2012) Available from: https://api.semanticscholar.org/CorpusID:125878838

[7] G.D. Birkhoff, A determinant formula for the number of ways of coloring a map, Annals Math., 14(1912)42-46, https://doi.org/10.2307/1967597

[8] H. Hosoya, On some counting polynomials in chemistry, Disc. Appl. math. (1988)239-257, http://dx.doi.org/10.1016/0166-218X(88)90017-0

[9] B.E. Sagan, Y.N. Yeh, P. Zhang, The Wiener polynomial of a graph. Int. J. of Quantum Chem., 60(1996)959-969, https://doi.org/10.1002/(SICI)1097-

[10] A.J. Schwenk, Computing the characteristic polynomial of a graph, In: Graphs and Combinatorics: Proceedings of the Capital Conf. on Graph Th. and Comb. at the George Washington Uni. 406(2006) 153-172, https://doi.org/10.1007/BFb0066438

[11] E. Deutsch, S. Klavžar, M-polynomial and degree-based topological indices. Iranian J. Math. Chem. 6(2015)93-102, https://doi.org/10.22052/ijmc.2015.10106

[12] S.A.K. Kirmani, P. Ali, J. Ahmad, Topological Coindices and Quantitative Structure Property Analysis of Antiviral Drugs Investigated in the Treatment of COVID-19, J. Chem. 2022(2022)1-15, https://doi.org/10.1155/2022/3036655

[13] A.H. Karim, N.E. Arif, A.M. Ramadan, The M-Polynomial and Nirmala index of certain composite graphs, Tikrit J. of Pure Sci., 27(2022)92-101, https://doi.org/10.25130/tjps.v27i3.45

[14] A.J.M. Khalaf, S. Hussain, D. Afzal, F. Afzal, A. Maqbool, M-polynomial and topological indices of book graph, J. Disc. Math. Sci. and Crypt., 23(2020)1217-1237, https://doi.org/10.1080/09720529.2020.1809115

[15] Z. Raza, M.E.K. Sukaiti, M-polynomial and degree based topological indices of some nanostructures, Symmetry, 12(2020)1-16, https://doi.org/10.3390/sym12050831

[16] B. Basavanagoud, A.P. Barangi, P. Jakkannavar, M-polynomial of some graph operations and cycle related graphs. Iran. J. Math. Chem., 10(2019)127-150, https://doi.org/10.22052/ijmc.2019.146761.1388

[17] A.A.K. Kirmani, P. Ali, CoM-polynomial and topological co-indices of Hyaluronic acid conjugates. Arab. J. Chem. 15(2022)111, https://doi.org/10.1016/j.arabjc.2022.103911

[18] M. Shanmukha, A. Usha, Comparative study of chitosan derivatives through CoM-polynomial, Int. J. Quantum Chem., 122(2022) e26976, https://doi.org/10.1002/qua.26976

[19] M. Shanmukha, S. Lee, A. Usha, K. Shilpa, M. Azeem, Structural descriptors of anthracene using topological coindices through CoM-polynomial, J. Intel. & Fuzzy Sys. 44(2023)8425-8436, https://doi.org/10.3233/JIFS-223947

[20] A. Jabeen, S. Ahmad, S. Zaman, The study of regression model based on com-polynomial in blood cancer drug properties, Partial Diff. Eq. in App. Math. 9(2024)100648, https://doi.org/10.1016/j.padiff.2024.100648

M-polynomial and CoM-polynomial of Hat-graphs

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المستخلص

يعتبر كثيرات M-polynomial وكثيرات CoM-polynomial من أهم كثيرات الحدود الرسومية المعتمدة على الدرجة، حيث يمكننا من خلالها الحصول على جميع المؤشرات الطوبولوجية والمؤشرات المشتركة المعتمدة على الدرجة تقريبًا. في هذه المقالة، سيتم حساب هذين كثيري الحدود لبعض الرسوم البيانية الجديدة المحددة من الرسوم البيانية الأساسية المعروفة، وستعرف هذه الرسوم البيانية باسم الرسوم البيانية Hat-graphs. تم إنشاء الرسوم البيانية Hat-graphs عن طريق إضافة رؤوس وحواف جديدة للرسوم البيانية الأساسية بطريقة خاصة. الرسوم البيانية الخاصة التي كانت أساس هذه الرسوم البيانية الجديدة هي Path، Cycle، Complete، Star، Wheel، Complete bipartite.