Study of the Radii and Deformation Parameters of Radioactive ($_{88}$Ra) Isotopes

Duaa Abed Salim

Department of Physics, College of Science, University of Mustansiriyah, Baghdad ,Iraq.

*Corresponding author E-mail: Duaa.A.salim@uomustansiriyah.edu.iq

https://doi.org/10.29072/basjs.20240208

ARTICLE INFO

Keywords

Radium, structure of nuclear , rotational symmetry.

ABSTRACT

This study examined many nuclear characteristics, including the composition, forms, and methods for altering the shapes of radioactive radium isotope nuclei based on the deformation parameters associated with these isotopes. The radii and deformation characteristics of these nuclei were also analyzed. The nuclear properties of radium require examination due to its common presence in the environment and its classification as an alpha emitter. Radium is a threat to natural ecosystems. The MATLAB software served as the operational tool for acquiring the results, in which several equations were executed to yield the desired outcomes. The results indicate that at mass number 230, β and δ get their peak values of 0.2692 and 0.2702, respectively, whilst at mass number 212, they achieve their lowest values of 0.0882 and 0.0704. Upon meticulous investigation, it was evident that there existed only a minor discrepancy between the experimental and acquired results, although the substantial differences in the outcomes. The nuclei exhibit spherical or nearly spherical structures and forms due of their elevated deformation parameters and near to the magic numbers. Consequently, their forms are inherently elliptical.

Received 1 May 2024; Received in revised form 17 Aug 2024; Accepted 22 Aug 2024, Published 31 Aug 2024

1. Introduction

Radium is a byproduct of uranium extraction that can be recovered from all uranium ores. Canada is home to uranium ores, which are the richest in radium [1]. Radium is currently removed from fuel rods used in nuclear reactors because the element is produced at a rate of less than 100 grams annually[2]. Radium 223 is the most significant isotope; it is occasionally used to treat prostate cancer that has recently migrated to the bones. Radium is utilized to target malignant bone cells since it belongs to the same group as calcium, which is found in bones[3]. It produces alpha particles, which are among the byproducts of radium's naturally occurring radioactive series and have the ability to destroy cancer cells[4].For instance, illuminating paints used on watches and clock dials contained radium. Alpha radiation is no longer deemed safe for usage, even though it cannot travel through glass or the metal of a watch case [5].

The most significant of these issues was characterizing the medium in the fundamental framework, which had an effect on the calculation of distorted nuclei and the structure of nuclear physics in the past[6]. Naturally, the spontaneous collapse of rotational symmetry is of interest to physicists [7]. Numerous characteristics of the nucleus seem to fluctuate or abruptly alter at specific even neutron or proton counts. Experimental observations have shown that the nucleus is stable when Z, the number of protons, or Z-A=N, the number of neutrons, reaches one of the following values: 126, 82, 50. 28, 20, 8, 2 [8].The motivation for this was that these numbers create closed, saturated shells that resemble the atom's electron shells. It seems that the neutron and proton shells are separate entities. It has been discovered that the weird magic numbers originate from the saturated neutron or proton shells[9].The nucleon-nucleon interaction is also referred to as the independent particle model since it is thought of as a weak interaction (weak pairing). The nucleons inside the nucleus are thought to rotate in their own orbits, with very little contact with one another that is independent of the other nucleons. The radius of each nucleon's orbit is established by the nuclear potential energy, according to the assumptions underlying the shell model. The process of gamma decay, or the emission of gamma rays from the nucleus with specific energies, supports the idea of nuclear orbits for nucleons but is the best evidence of the strength and sobriety of the shell model. The radius of its orbit is determined by the nuclear potential energy that results from the interaction or attraction of the nucleon with the other nucleons[10].

This section will include some significant investigations that involved theoretical calculations, such as root-mean-square and deformation parameter calculations, due to the characteristics of particular isotopes. We will also discuss the findings from these investigations. In 2022, the experimental data on the average transparent charge diameter of the atomic nucleus were examined by M. V. Chushnyakova and others. The study's conclusions included a clearer understanding of how neutron shells arise from what is observable and the regularity with which the charge radius is controlled at the neutron. Thus, the analysis demonstrated that, although the missiles did not detect neutrons 20 and 126, they did record their presence. This number is obtained as the ratio of the charge radius of an isotope to the charge radius of that isotope of the same element with a closed neutron shell [11].

In 2019, Eriba-Idoko F. performed investigations with the initial excited state energies of the nuclei. the nuclear deformation parameters β2, the reduced quadruple transition probability B(E2)↑, and other inherent characteristics related to the nuclei shape. The results indicated that compared to the nuclei of the Yb isotopes, the Pb nucleus, which has a Z = 82, one of the "magic numbers," has a more "spherically" formed nucleus at the ground state with less distortion [12].

In 2022, copper and its isotopes were employed by researchers R. An, X. Jiang, L. Cao, and F. Zhang in a thorough investigation of the radius of the root mean square of the charge. It was believed that these computations were essential for establishing connections between protons and neutrons. The trial's results were contrasted. Using NL3, PK1, and NL3, the study's results were provided using the relative mean field (RMF) model. Some of the sharp changes across N = 50 and at 82 clearly showed that the shell of the nuclear charge had closed. Another method to describe the behavior of the r.m.s charge radius is as an inverted parabola between two neutron magic numbers (N = 28 to 50 for copper isotopes and N = 50 to 82 for indium isotopes [13]. Radium was not covered in the aforementioned studies. Similarly, the extent to which deformation affects the nuclei and the morphologies of the nuclei were not examined in earlier research. The technique employed in this study proved excellent at modeling the shapes of isotopes and visualizing nuclei. Furthermore, because the simulation was carried out in a manner different from that handled by prior studies, the radii of the even-even nuclei were also analyzed with nuclear properties that had not been studied previously. This research's conclusions were computed differently from those of previous studies.

The study's aim and the methodological changes in the nuclear charge radius are considered to be extremely important nuclear aspects. Neutron-proton correlations are crucial for quantitatively determining the shapes, deformations, and structures of the nuclei, as they rely on the radius of the nuclear charge along the even isotope chains Z and N. Moreover, results for the element radium were significantly different from what we had concluded. Owing to its extensive distribution in wall paint and construction supplies, it is an essential part of our daily lives[14].

2. Methodology

In this section, we will go over the work process in detail step by step. In the first stage, radium-related data was gathered from libraries [15]. In stage two, the information was manually input. Following that, MATLAB was used to program a few equations. Subsequently, a unique plotting technique was employed to ascertain the nuclei's forms and the degree of deformation in those shapes.

Nuclear shapes are typically spherical when nuclei are stable. nevertheless, the effort was to lower the surface energy. Only the ratio can be used to measure and monitor deformations and small portions of these fields in the region 150<A<190 [15].

$$\delta = \Delta R/R \qquad (1)$$

R is the nuclear radius average

ΔR is the difference between semi- minor (a) and semi- major (b) axes.

$$\Delta R = (b - a) \qquad (2)$$

It has been possible to interpret the rotations and vibrations on the nuclear surface as the collective movement in the collective model by using the geometric model put out by scientists Bohr and Motelson [16]. The nucleus created by extension can be represented as a charged liquid droplet, and the nuclear surface transformation in a spherical shape essentially corresponds to dependent on time shape parameters that are considered parameters [17, 18]. The quadruple deformation parameter β_2 and the spherical axis have a relationship, as seen by the equation shown following [19] [20]:

$$\beta_2 = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}} = 1.06\frac{\Delta R}{R_{av}} \qquad (3)$$

Where:

The average radius $R_{av} = R_o A^{(1/3)}$.

Important details about the primary nuclear materials that represent the efficiency of the ordered nuclear structure are provided by the root mean square (rms) radius of the nuclear charge and other aspects of the nuclear stable ground state; for instance, the shell closes and starts to deform. [21]. where the charge distribution radius, 〈r^2〉 , is the square of the root mean square (rms) radius, which is inferred from the distribution of scattered electrons.

$$< r^2 >= \frac{\left[0.63\, R_o^2\, (1 + \frac{10}{3}(\frac{\pi\, a_o}{R_o})^2\right]}{\left[1 + (\frac{\pi a_o}{R_o})^2\right]} \qquad A \leq 100 \qquad (4)$$

Where:

A: Mass number , R is the radius of the nuclei .

The charge distribution within the nucleus can be explained by the multiple electrical moments [22]. It is possible to measure the quadruple moment of several nuclei experimentally. It is anticipated that these nuclei will have an axial symmetry and an ellipsoid shape. The following classical expression [23] defines the fundamental quadruple moment:

$$Q_0 = \frac{4}{5} Z R^2 \delta \qquad (5)$$

The nucleus quadruple parameter deformation values δ calculated from the equation[24]:

$$\delta = 0.75\, Q_0/(Z\langle r^2 \rangle) \qquad (6)$$

The semi-axes (a) and (b) are gained from the two following equations [25].

$$a = \sqrt{(\langle r^2 \rangle (1.66 - \frac{2\delta}{0.9})} \qquad (7)$$

$$b = \sqrt{5\langle r^2\rangle - 2a^2} \qquad (8)$$

3. Results and discussion

3.1 Results

This paper is a systematic investigation of various deformation parameters (β) and (δ) of the nuclear charge radius through radium isotope chains. where the particular experimental mass determines the coupling strength [15]. While the deformation parameter (β) was determined using the previously reported equation (3), the results of the equation (6) for radium isotopes are in agreement with the experimental results [15]. The radium isotopes' (rms) charge radius is calculated from the modified results obtained from equation (4) for even-even nucleus.

Table1: Theoretical values and present study of probability of electric transition , and parameters of deformation of (Ra-88) isotopes.

Z	A	N	E_γ (KeV)	Present work				
				β for *SSANM*	R_0 (fm)	Q_o *(b)*	β	δ
88	212	124	642.90	….	6.0334	3.0078	0.0882	0.0704
	214	126	1382.4	….	6.0503	2.0230	0.0589	0.0471
	216	128	688.20	….	6.0670	2.8583	0.0828	0.0662
	218	130	389.10	0.095	6.0837	3.7897	0.1091	0.0873
	220	132	178.47	….	6.1003	5.5786	0.1596	0.1278
	222	134	111.12	0.192	6.1168	7.0486	0.2004	0.1606
	224	136	84.373	0.1790	6.1332	8.0649	0.2280	0.1827
	226	138	67.67	0.2022	6.1495	8.9788	0.2523	0.2024
	228	140	63.823	0.217	6.1657	9.2183	0.2575	0.2067
	230	142	57.4	….	6.1818	9.6922	0.2692	0.2162
	232	144	97.700	….	6.1978	7.4076	0.2045	0.1644

Table2 : Root mean square radii $< r^2 >^{(\frac{1}{2})}$ and minor axes (a) and major axes (b) , the (Δ R) between axes (a and b) by using two method for (Ra-88) isotopes.

Z	A	N	*Present study*					
			$\langle r^2 \rangle^{1/2}$ fm	a (fm)	b (fm)	Δ R_1 (fm)	Δ R_2 (fm)	Δ R_3 (fm)
88	212	124	0.5120	3.0185	3.4560	0.4493	0.4375	0.5971
	214	126	0.5152	3.0742	3.3690	0.3015	0.2949	0.4003
	216	128	0.5184	3.0364	3.4491	0.4249	0.4127	0.5639
	218	130	0.5216	2.9933	3.5354	0.5620	0.5421	1.7454
	220	132	0.5248	2.9043	3.6920	0.8253	0.7877	1.0939
	222	134	0.5280	2.8306	3.8157	1.0403	0.9851	1.3780
	224	136	0.5311	2.7806	3.8991	1.1874	1.1185	1.5719
	226	138	0.5343	2.7357	3.9724	1.3189	1.2367	1.7449
	228	140	0.5374	2.7285	3.9924	1.3509	1.2640	1.7862
	230	142	0.5406	2.7081	4.0301	1.4171	1.3221	1.8725
	232	144	0.5437	2.8401	3.8545	1.0806	1.0144	1.4270

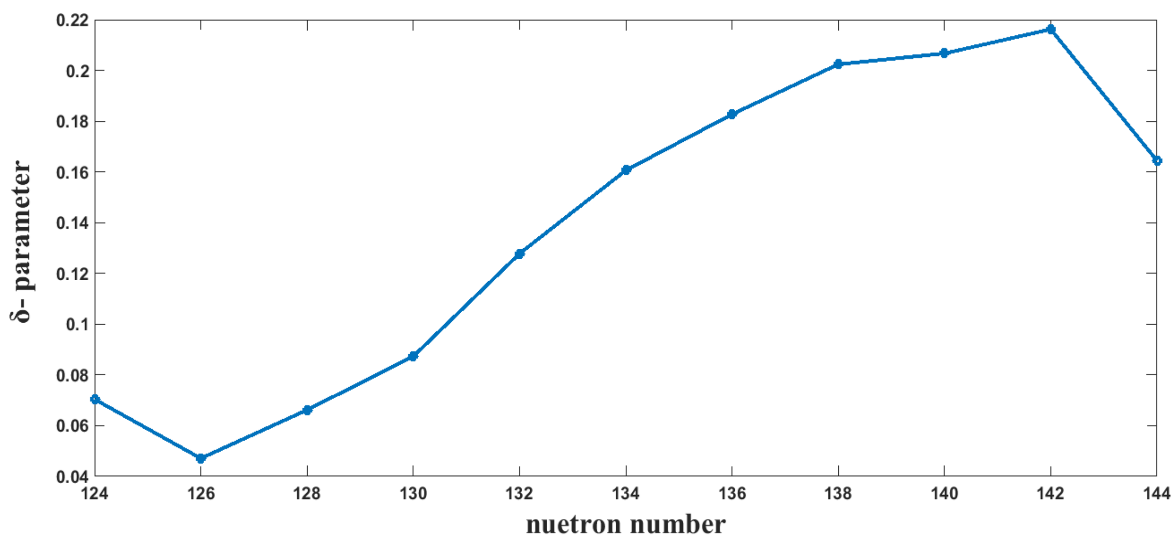


Figure 1. δ deformation with neutron number .

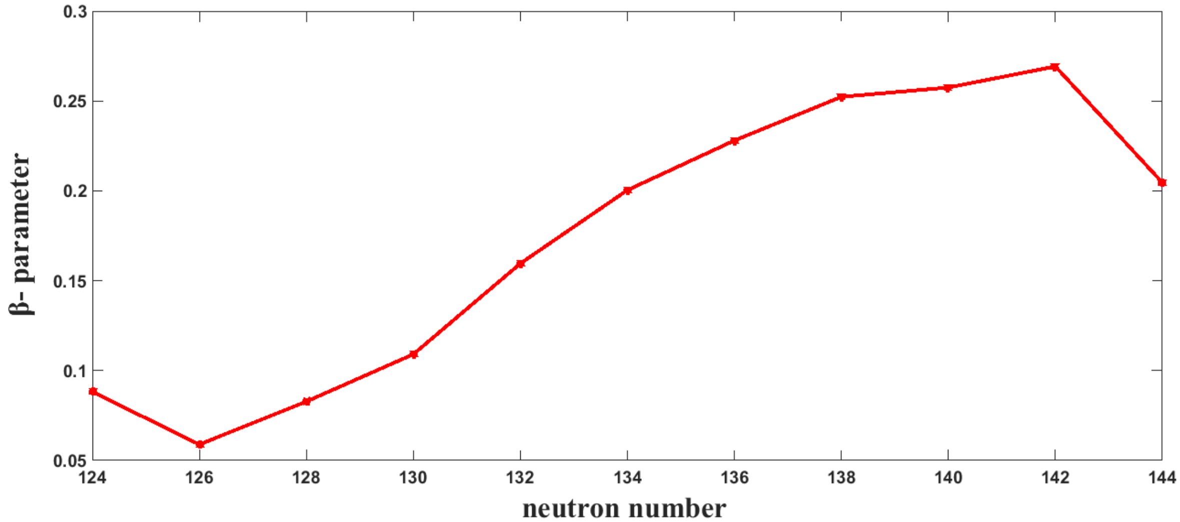
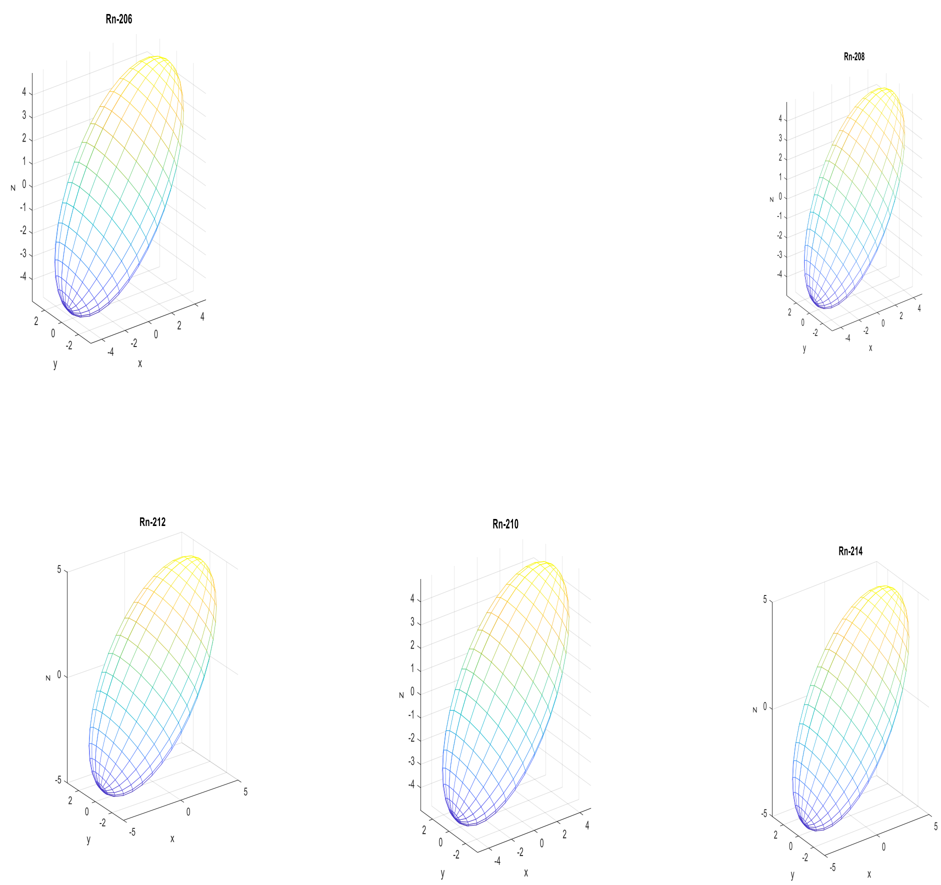


Figure 2. β deformation with neutron number



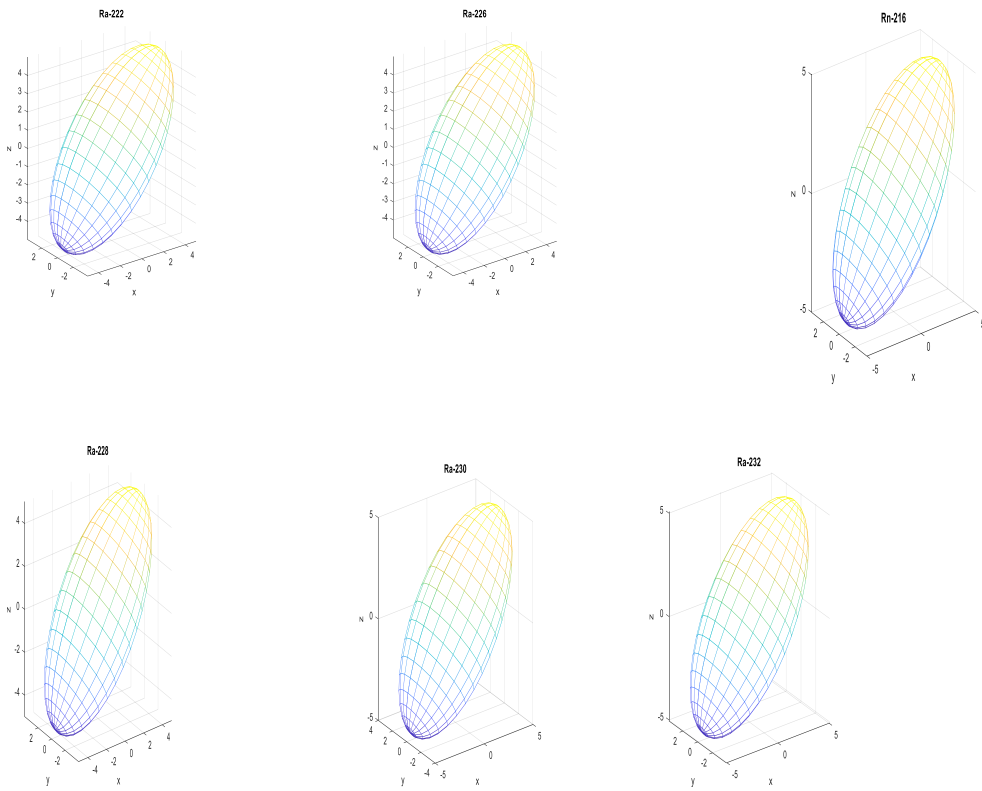


Figure 3. Three-dimensional figures demonstrating the deformation of the nuclei.

3.2 Discussions

The estimated distortion parameters in Table1 produced remarkably unique data, particularly when the isotopes' neutron number is very near to the magic number (126). The results show that the number of neutrons decreases with the deformation parameters (δ and β), and then the number of neutrons increases suddenly with the deformation parameters. For example, at neutron number 124, the value of (β) is (0.0882). From there, it increases gradually until it reaches (0.2045) at (144). The value of δ , on the other hand, also starts to rise gradually and eventually reaches 0.1644. Additionally, a comparison was conducted with the beta values and the experimental results[12], as indicated in Table 1. Table 2 illustrates the results of the calculations made using equation (4) for the difference between the major axes and the average radius of the mass numbers, both of which range in values between (0.5120–0.5437), which is larger than 100. equations (7) and (8) were also used to calculate the two main axes.

The deformation parameter (β) and neutron number are plotted on the y- and x-axes, respectively, in Figure 1. There is just one value at the neutron number 126 since it has less distortion because it is near the magic number. However, there is a progressive increase between them, indicating a direct relationship. Figure 2 displays the neutron number on the x-axis and the deformation parameter (δ) on the y-axis. This chart demonstrates their direct connection, with an increase in the distortion parameter corresponding to an increase in the neutron number. The three-dimensional shapes are displayed in Figure 3. Since these structures were created using the main axes, this figure illustrates the amount of curvature in the nuclei.

4. Conclusions

Some of the radium isotopes' nuclear characteristics were examined in this study. Radium is one of the most significant elements that has to be studied because of its many applications, significance, and degree of danger. It is also one of the nuclear chains that produces alpha rays, which are nuclear emissions from it. The investigation ultimately arrived at the conclusion that this element's nuclei are undergoing distortions and that there is a direct correlation between the neutron number and the distortion parameters, also known as the quadruple moment. Additionally, it was determined that the distortion in the nucleus will be reduced if one of the nucleons had a magic number. This indicates that it typically Its spherical nucleus makes it more stable.

References

[1] G.M. Mudd, The future of Yellowcake: A global assessment of uranium resources and mining, Sci.of the Env., 472(2014). 590-607. https://doi.org/10.1016/j.scitotenv.2013.11.070

[2] S.Soniya, J. P.John, M. Khandaker, Study of radium content and radon exhalation rates in raw building materials used in southern India. Phys.Op., 17(2023) 3, https://doi.org/10.1016/j.physo.2023.100169

[3] A., Ch. K. Hemathilaka, W.R.Chuanchuan., Prostate Cancer, Castration-Resistant Prostate Cancer (CRPC), Radium-223 Dichloride Injection for Bone Metastasized Prostate Cancer, J Can. Ther.,. 1411(2023) 429-442, https://doi.org/10.4236/jct.2023.1411036

[4] G. Kolarević, O. Arsovski,B. Predojević,Twelve decades of using radium in the treatment of deeper localized cancers, Scr. Med., 544 (2023) 419-424. https://doi.org/10.5937/scriptamed54-45969

[5] Y. Osika , S. Sharashkin , G. Pitsevich , M. Shundalau, initio potential energy surfaces and spectroscopic and radiative properties of the low-lying states of the radium monohydroxide RaOH radical, J. Quant. Spectrosc. Radiat. Transf., 314 (2024) 108-852. https://doi.org/10.1016/j.jqsrt.2023.108852

[6] E. Caurier, J. Egido, G. Martínez, A. Poves, J. Retamosa, L. Robledo, A. Zuker, Laboratory frame description of the deformed nucleus 48Cr, Phys. Rev. Lett., 75 (1995) 24-66, https://doi.org/10.1103/PhysRevLett.75.2466

[7] B. Brown, W. Rae, The shell-model code NuShellX@MSU, Nucl. Data Sheets, 120 (2014) 115-118, https://doi.org/10.1016/j.nds.2014.07.022

[8] R. Y. Nuwayhid, The essential physics of thermoelectricity for mechanical engineers, Int. J. Mech. Eng. , 304(2002) 351-368., https://doi.org/10.7227/IJMEE.30.4.7

[9] M. Pfützner , M. Karny Radioactive decays at limits of nuclear stability, Rev. Modren Phys., 842(2012)567-619, https://doi.org/10.1103/RevModPhys.84.567

[10] B. Cederwall, F. Gh. Moradi, T. Bäck, A. Johnson, J. Blomqvist, E. Clément, G. de France, R. Wadsworth, K. Andgren, K. Lagergren, A. Dijon, G. Jaworski, R. Liotta, C. Qi, B. M. Nyakó, J. Nyberg, M. Palacz, H. Al-Azri, A. Algora, G. de Angelis, A. Ataç, S. Bhattacharyya, T. Brock, J. R. Brown, S. Williams, Evidence for a spin-aligned neutron–proton paired phase from the level structure of ^{92}Pd, Natue, 469 (2011) 68-71. https://doi.org/10.1038/nature09644

[11] E. V. Vladimirova, I. D. Dashkov, B. S. Ishkhanov , T. Yu. Tretyakova, Mass Ratios and Proton Pairing for Isotones with N = 20, 50, and 82, Bull. Russ. Acad. Sci. Phys., 82 (2018) 685–690, https://doi.org/10.3103/S1062873818060345

[12] F. E.Idoko, T. Daniel, F. Gbaorun, E. C. Hemba, Nuclei Shapes across Different Magic Numbers in the Ytterbium (Z= 70) and Lead (Z= 82) Isotopes using MATLAB Code, Nig Ann Pure App Sci., 2(2019)75-86, https://doi.org/10.46912/napas.111

[13] R.An, X. Jiang, L.G. Cao, F.S. Zhang ,Evolution of nuclear charge radii in copper and indium isotopes, Chin. Phys. C, 6 (2022) 64-101, https://doi.org/10.1088/1674-1137/ac501a

[14] D.B.Chambers, Radiological protection in North American naturally occurring radioactive material industries, ICRP., 441(2015)202-213. https://doi.org/10.1177/0146645315572300

[15] S. Raman ,C.W.Nestron , P.tikkanen, Transition probability from the ground to the first-excited 2+ state of even–even nuclides, At. Data Nucl. Data Tables., 781(2001)1-128. https://doi.org/10.1006/adnd.2001.0858

[16] O.Hen, G. A. Miller, E. Piasetzky, L. B. Weinstein, Nucleon-nucleon correlations, short-lived excitations, and the quarks within, Rev. Modren Phys., 4(2017) 4-89. https://doi.org/10.1103/RevModPhys.89.045002

[17] D.A.Salim, S.A. Ebrahiem , Study of transition strength for two isotopes carbon and oxygen, AIP Conf. Proc., 2437 (2022)1, https://doi.org/10.1063/5.0093768

[18] A. Al-Sayed , A. Y. Abul-Magd, Level statistics of deformed even-even nuclei, Phys. Rev, 3(2006) 3-74 , https://doi.org/10.1103/PhysRevC.74.037301.

[19] N. T. Jarallah , Quadrupole Moment and Deformation Parameter for Even-Even 38Sr (A= 76-102) Nuclide, En. Proceda, 157 (2019) 276-282. https://doi.org/10.1016/j.egypro.2018.11.191

[20] S. Cho, T. Hyodo, D. Jido , Ch. Ko , S.H. Lee, S. Maeda , K. Miyahara , K. Morita , M. Nielsen , A. Ohnishi , T. Sekihara , T.Song , S. Yasui , K.Yazaki, Exotic hadrons from heavy ion collisions, Prog. Part. Nucl. Phys., 95 (2017) 279-322, https://doi.org/10.1016/j.ppnp.2017.02.002

[21] S.A.Ibrahim, S. M. Ibrahim, A. B. Rhaiem, Calculating of the Electric Transition Forces and Radii of Even-even-Nuclei of Cadmium (100-12448Cd) Cd Isotopes, Ibn AL-Haitham J. Pure Apll. Sci., 372 (2024)217-225, https://doi.org/10.30526/37.2.3468

[22] G. Neyens, Nuclear magnetic and quadrupole moments for nuclear structure research on exotic nuclei, Rep. Prog. Phys., 4(2003) 664-633. https://doi.org/10.1088/0034-4885/66/4/205

[23] S. Mohammadi,"Quadrupole Moment Calculation of Deformed Nuclei, J. Phys.Conf. Series, 381(2012) 12-129, https://doi.org/10.1088/1742-6596/381/1/012129

[24] I. Boboshin, B. Ishkhanov, S. Komarov, V. Orlin, N. Peskov ,V. Varlamov, Investigation of quadrupole deformation of nucleus and its surface dynamic vibration, Int. Conf. Nucl Data Sci. Technolo., 4(2007) 65-68. https://doi.org/10.1051/ndata:07103

[25] A.A. Ridha, Deformation parameters and nuclear radius of Zirconium (Zr) isotopes using the Deformed Shell Model, J Wasit Sci. Med., 21(2009)115-125. https://doi.org/10.31185/jwsm.35

دراسة انصاف اقطار و معلمات التشوه للنظائر المشعة لعنصر ($_{88}$Ra)

دعاء عبد سالم

قسم الفيزياء، كلية العلوم، الجامعة المستنصرية، بغداد، العراق

المستخلص

تم في هذه الدراسة فحص العديد من الخصائص النووية منها معلمات التشوه وأنصاف أقطار نوى نظائر الراديوم المشعة بالإضافة الى ذلك دراسة اشكالها وتركيبها وكيفية تغيير اشكالها تبعا لمعلمات التشوه الحاصلة في هذه النوى . أن الراديوم متوفر بكثرة في البيئة وبما انه يشكل خطرا على الحياة الطبيعية والذي يعتبر احد بواعث الفا فهو أحد العناصر التي يجب دراسة خصائصها النووية . تم استخدام برنامج MATLAB كأداة عمل للحصول على النتائج ، حيث تمت برمجة بعض المعادلات والاستفادة منها للحصول على النتائج التي تم الحصول عليها . تشير النتائج إلى أن β و δ لهما قيم القصوى والتي هي 0.2692 و0.2702 عند العدد الكتلي 230، على التوالي، والقيم الدنيا 0.0882 و0.0704 عند العدد الكتلي 212. وبعد دراسة مستفيضة، تم اكتشاف أن هناك اختلافًا بسيطًا جدًا بين النتائج التجريبية والنتائج التي تم الحصول عليها , حيث كانت نتائج مميزة للغاية. بالتالي يشير هذا إلى أنه مع زيادة العزم الرباعي وزيادة احتمالات الانتقال من المستوى المثار الأولي، تنخفض معلمات التشوه بالاعتماد على العزم الرباعي الجوهري عند الاعداد السحرية لذلك فان اشكال النوى وتراكيبها تكون كروية او قريبة من الشكل الكروي ، أو كلما اقتربت القيم من الاعداد السحرية لذلك فانه يؤدي إلى ارتفاع معلمات التشوه لذلك اشكالها تميل الى الشكل البيضوي .