Rishi Transform to Solve Population Growth and Decay Problems

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ABSTRACT

This study aims to explore the application of the Rishi transform to population growth and decay problems, which are pivotal in various fields such as finance, physical science, chemistry, biology, and sociology. The primary objectives are to demonstrate the effectiveness of the Rishi transform in solving these problems and to provide several practical applications that highlight its value and utility. The results confirm the Rishi transform's capability and usefulness in addressing population growth and decay issues.

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1. Introduction

Mathematical models are crucial for improving that how accurate is the biotechnological process, and differential equations are often employed in many branches of science and engineering to represent complicated physical events [1, 2]. Integral transforms are currently regarded among the significant mathematical methods to find solutions of complex problems in science, technology, engineering and finance because of its important characteristic which convert a complicated problem to a simpler one. The potential of integral transforms to give the exact solutions of problems without requiring lengthy calculations is a crucial feature and advantage. Investigating new integral transforms are of much interest of researchers, but the application of these transforms in diverse fields, equations in different domains, are also significant and considered as aim of researchers. Very recently, Hozan Hilmi, et al [3] used Sawi transform and sequential approximation method for exact and approximate solution of Multi-Higher order fractional differential equations. Then, Aggarwal et al [4] applied Mahgoub transform to solve growth and decay problems. Moreover, Saadeh et al [5] used double formable integral transform for solving heat equations via using numerical examples. Güngör [6] applied Kharrat-Toma transform for solving linear Volterra integral equations. A new transform which is called Rishi transform introduced by R.Kumar et al [7] that also used to solve multi-higher order fractional differential equations by Ali Turab et al [8]. In our study, Rishi transform is used to find the analytical solutions of differential equations of growth and decay problems. The paper is organized as following, Section two contain preliminaries. In section three Rishi transform is utilized for problems of growth and decay. Applications are drawn analytically and graphically in section four and the last section gives the conclusion.

2. Preliminaries

This section, present the definition of Rishi transform m and provide the preliminary concepts required to solve the growth and decay problems.

Definition 1: [7, 8] Rishi transform defines the exponentially-order piecewise continuous function $y(t)$ on the interval $[0,\infty)$ as follows:

$$\mathcal{R}\{y(t)\} = Y(\varepsilon,\sigma) = \left(\frac{\sigma}{\varepsilon}\right)\int_0^\infty y(t)e^{-\left(\frac{\varepsilon}{\sigma}\right)t}dt,\ \ \varepsilon > 0, \sigma > 0\ and\ t \geq 0$$

Also, the inverse Rishi transform is:

$$y(t) = \mathcal{R}^{-1}[Y(\sigma,\varepsilon)] = \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)\int_0^\infty y(t)e^{-\left(\frac{\varepsilon}{\sigma}\right)t}dt\right]$$

the $\mathcal{R}^{-1}$will be the inverse of the $\mathcal{R}$ Rishi transform.

Remark: According to the definition, it can be applied to some continuous functions are as follows [8-10] :

i. Apply the Rishi transformation for some fundamental functions is as follows:

$y(t), t > 0$	$\mathcal{R}\{y(t)\} = Y(\varepsilon,\sigma)$	$y(t), t > 0$	$\mathcal{R}\{y(t)\} = Y(\varepsilon,\sigma)$
1	$\left(\frac{\sigma}{\varepsilon}\right)^2$	$sinrt$	$\frac{r\sigma^3}{\varepsilon(\varepsilon^2+r^2\sigma^2)}$
e^{rt}	$\frac{\sigma^2}{\varepsilon(\varepsilon-r\sigma)}$	$cosrt$	$\frac{\sigma^2}{(\varepsilon^2+\sigma^2r^2)}$
$t^n, n \in \mathbb{N}$	$n!\left(\frac{\sigma}{\varepsilon}\right)^{n+2}$	$sinhrt$	$\frac{r\sigma^3}{\varepsilon(\varepsilon^2-r^2\sigma^2)}$
$t^\alpha, \alpha > -1, \alpha \in \mathbb{R}$	$\Gamma(\alpha+1)\left(\frac{\sigma}{\varepsilon}\right)^{\alpha+2}$	$coshrt$	$\frac{\sigma^2}{(\varepsilon^2-\sigma^2r^2)}$

ii. Apply inverse the Rishi transformation for some fundamental functions is as follows:

$Y(\varepsilon,\sigma)$	$\mathcal{R}^{-1}\{Y(\varepsilon,\sigma)\} = y(t)$	$Y(\varepsilon,\sigma)$	$\mathcal{R}^{-1}\{Y(\varepsilon,\sigma)\} = y(t)$
$\left(\frac{\sigma}{\varepsilon}\right)^2$	1	$\frac{r\sigma^3}{\varepsilon(\varepsilon^2+r^2\sigma^2)}$	$sinrt$
$\frac{\sigma^2}{\varepsilon(\varepsilon-r\sigma)}$	e^{rt}	$\frac{\sigma^2}{(\varepsilon^2+\sigma^2r^2)}$	$cosrt$
$n!\left(\frac{\sigma}{\varepsilon}\right)^{n+2}$	$t^n, n \in \mathbb{N}$	$\frac{r\sigma^3}{\varepsilon(\varepsilon^2-r^2\sigma^2)}$	$sinhrt$
$\Gamma(\alpha+1)\left(\frac{\sigma}{\varepsilon}\right)^{\alpha+2}$	$t^\alpha, \alpha > -1, \alpha \in \mathbb{R}$	$\frac{\sigma^2}{(\varepsilon^2-\sigma^2r^2)}$	$coshrt$

iii. Linearity property of Rishi transform and inverse Rishi transform.

If $\mathcal{R}\{y(t)\} = Y(\varepsilon,\sigma)$ and $\mathcal{R}\{x(t)\} = X(\varepsilon,\sigma)$, then

$$\mathcal{R}\{ay(t) + bx(t)\} = a\mathcal{R}\{y(t)\} + b\mathcal{R}\{x(t)\} = aY(\varepsilon,\sigma) + bX(\varepsilon,\sigma)$$

Also relative to the inverse Rishi transform, it will be as follows:

If $y(t) = \mathcal{R}^{-1}[Y(\varepsilon, \sigma)]$ and $x(t) = \mathcal{R}^{-1}[X(\varepsilon, \sigma)]$, then

$$ay(t) + bx(t) = a\mathcal{R}^{-1}[Y(\varepsilon, \sigma)] + b\mathcal{R}^{-1}[X(\varepsilon, \sigma)],$$

where a, b are arbitrary constant.

Property 1: [8] For the integer order derivative of $y(t)$, the Rishi transformation is:

$$\mathcal{R}\{y^{(m)}(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^{m} Y(\varepsilon, \sigma) - \sum_{k=0}^{m-1} \left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0) \qquad (1)$$

3. Applying Rishi transform in problems of population growth and decay

Population growth model

We can mathematically describe the population expansion of a plant, cell, organ, or species by using first order ordinary linear differential equation [11-14] as

$$\frac{dy}{dt} = ky \qquad (2)$$

With initial condition $y(t_0) = y_0$, such that $k \in R^{+}$, y is the amount of people living at time t and y_0 is the original population at $t = t_0$.

Equation (2) represents the population growth Malthusian law.

The following first order ordinary linear differential equation mathematically define decay problem of the substance [14] as

$$\frac{dy}{dt} = -ky \qquad (3)$$

with initial condition $y(t_0) = y_0$, where y is the substance amount at time t, $k \in R^{+}$ and y_0 is the initial substance amount at $t = t_0$.

The negative sign in the R.H.S of (3) is taken as the substance mass is declining over time, then $\frac{dy}{dt}$ should be negative.

A. Rishi transform for problem of population growth

We introduce the Rishi transform for problem of the population increase in this part. Its mathematical formulation is presented by (2)

Taking Rishi transform on each side of (2), we obtain

$$\mathcal{R}\left\{\frac{dy}{dt}\right\} = k\mathcal{R}\{y(t)\}$$

Now employing of the property 1, Rishi transform of derivative of function, on (2), we obtain

$\mathcal{R}\{y^{(m)}(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^m Y(\varepsilon,\sigma) - \sum_{k=0}^{m-1} \left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0)$, we have $m = 1$

So, we obtain $\frac{\varepsilon}{\sigma}Y(\varepsilon,\sigma) - \sum_{k=0}^{m-1} \left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0) = k\mathcal{R}\{y(t)\}$

$$\frac{\varepsilon}{\sigma}Y(\varepsilon,\sigma) - \frac{\sigma}{\varepsilon}y(0) = kY(\varepsilon,\sigma) \qquad (4)$$

Using initial condition $y(t_0) = y_0$ in (4) and on simplification, we have

$$\left(\frac{\varepsilon}{\sigma} - k\right)Y(\varepsilon,\sigma) = \frac{\sigma}{\varepsilon}y_0 , \Rightarrow (\varepsilon - k\sigma)Y(\varepsilon,\sigma) = \frac{\sigma^2 y_0}{\varepsilon} \Rightarrow Y(\varepsilon,\sigma) = \frac{\sigma^2 y_0}{\varepsilon(\varepsilon-k\sigma)} \qquad (5)$$

Operating inverse Rishi transform on each side of (5), we obtain

$$\mathcal{R}^{-1}\{Y(\varepsilon,\sigma)\} = \mathcal{R}^{-1}\left\{\frac{\sigma^2 y_0}{\varepsilon(\varepsilon - k\sigma)}\right\} \Rightarrow y(t) = \mathcal{R}^{-1}\left\{\frac{\sigma^2 y_0}{\varepsilon(\varepsilon - k\sigma)}\right\}$$

$$y(t) = y_0\mathcal{R}^{-1}\left\{\frac{\sigma^2}{\varepsilon(\varepsilon - k\sigma)}\right\} \Rightarrow y(t) = y_0 e^{kt} \qquad (6)$$

that is the population amount required at time t.

B. Rishi transform for decay problem

This section, shows Rishi transform for problem of decay which is expressed mathematically in (3).

Employing the Rishi transform on each side of (3), we obtain

$$\mathcal{R}\left\{\frac{dy}{dt}\right\} = -k\mathcal{R}\{y(t)\} \qquad (7)$$

Now employing the property 1, Rishi transforms of derivative of function, on (7), we obtain

$$\frac{\varepsilon}{\sigma}Y(\varepsilon,\sigma) - \sum_{k=0}^{m-1}\left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0) = -k\mathcal{R}\{y(t)\}$$

$$\frac{\varepsilon}{\sigma}Y(\varepsilon,\sigma) - \frac{\sigma}{\varepsilon}y(0) = -kY(\varepsilon,\sigma) \quad (8)$$

Using initial condition $y(t_0) = y_0$ in (8) and on simplification, we have

$$\left(\frac{\varepsilon}{\sigma} + k\right)Y(\varepsilon,\sigma) = \frac{\sigma}{\varepsilon}y_0\,, \Rightarrow (\varepsilon + k\sigma)Y(\varepsilon,\sigma) = \frac{\sigma^2 y_0}{\varepsilon} \Rightarrow Y(\varepsilon,\sigma) = \frac{\sigma^2 y_0}{\varepsilon(\varepsilon+k\sigma)} \quad (9)$$

Operating inverse Rishi transformation on both sides of (9), we have

$$\mathcal{R}^{-1}\{Y(\varepsilon,\sigma)\} = \mathcal{R}^{-1}\left\{\frac{\sigma^2 y_0}{\varepsilon(\varepsilon+k\sigma)}\right\} \Rightarrow y(t) = \mathcal{R}^{-1}\left\{\frac{\sigma^2 y_0}{\varepsilon(\varepsilon+k\sigma)}\right\} \Rightarrow y(t) = y_0\mathcal{R}^{-1}\left\{\frac{\sigma^2}{\varepsilon(\varepsilon+k\sigma)}\right\}$$

$$\Rightarrow y(t) = y_0 e^{-kt} \quad (10)$$

That is the population amount required at time t.

4. Applications

The benefit of the Rishi transforms for issues related to population growth and decay is illustrated in a few cases provided in this section. In this work the applications are same with [14-17] except that we changed the hypothetical values and illustrated them graphically. Also we illustrated figures depending on the data. If we compare these figures with the analytical solutions, we obtain the same result which makes this method more applicable.

Application 1: A city's population increases at a rate that is proportionate to the total number of residents already residing there. If five years later, the population has grown by twofold and seven years later the population are 40,000, calculate the initial number of people living in the city.

Solution: Mathematically, the problem might be expressed as:

$$\frac{dy}{dt} = ky \quad (11)$$

where y is the total population of the city at time t and k is the proportionality constant. Regarding y_0 as the original population living in the city at $t = 0$.

Now employing property 1, Rishi transforms of derivative of function, on (11), we obtain

$$\mathcal{R}\{y^{(m)}(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^m Y(\varepsilon,\sigma) - \sum_{k=0}^{m-1} \left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0), \text{ we have } m = 1$$

So we obtain $\frac{\varepsilon}{\sigma} Y(\varepsilon,\sigma) - \sum_{k=0}^{m-1} \left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0) = k\mathcal{R}\{y(t)\}$

$$\frac{\varepsilon}{\sigma} Y(\varepsilon,\sigma) - \frac{\sigma}{\varepsilon} y(0) = kY(\varepsilon,\sigma)$$

Using initial condition $y(t_0) = y_0$ in (4) and on simplification, we have

$$\left(\frac{\varepsilon}{\sigma} - k\right) Y(\varepsilon,\sigma) = \frac{\sigma}{\varepsilon} y_0 \,, \Rightarrow (\varepsilon - k\sigma)Y(\varepsilon,\sigma) = \frac{\sigma^2 y_0}{\varepsilon} \Rightarrow Y(\varepsilon,\sigma) = \frac{\sigma^2 y_0}{\varepsilon(\varepsilon - k\sigma)}$$

Operating inverse Rishi transformation on both sides of above equation, we obtain

$$\mathcal{R}^{-1}\{Y(\varepsilon,\sigma)\} = \mathcal{R}^{-1}\left\{\frac{\sigma^2 y_0}{\varepsilon(\varepsilon - k\sigma)}\right\} \Rightarrow y(t) = \mathcal{R}^{-1}\left\{\frac{\sigma^2 y_0}{\varepsilon(\varepsilon - k\sigma)}\right\}$$

$$y(t) = y_0 \mathcal{R}^{-1}\left\{\frac{\sigma^2}{\varepsilon(\varepsilon - k\sigma)}\right\} \Rightarrow y(t) = y_0 e^{kt} \qquad (12)$$

Now at $t = 4, y = 2y_0$, so using this in (12), we have

$$2y_0 = y_0 e^{5k} \Rightarrow e^{5K} = 2$$

$$\Rightarrow k = 0.2ln2 = 0.1386 \qquad (13)$$

Now using the condition at $t = 7, y = 40{,}000$, in (12), we obtain

$$40{,}000 = y_0 e^{7k} \qquad (14)$$

Replacing the value of k from (13) in (14), we obtain

$$40{,}000 = y_0 e^{7\times 0.1386} \Rightarrow 40{,}000 = 2.639 y_0 \Rightarrow y_0 \cong 15{,}156$$

which are the population required who living in the city originally.

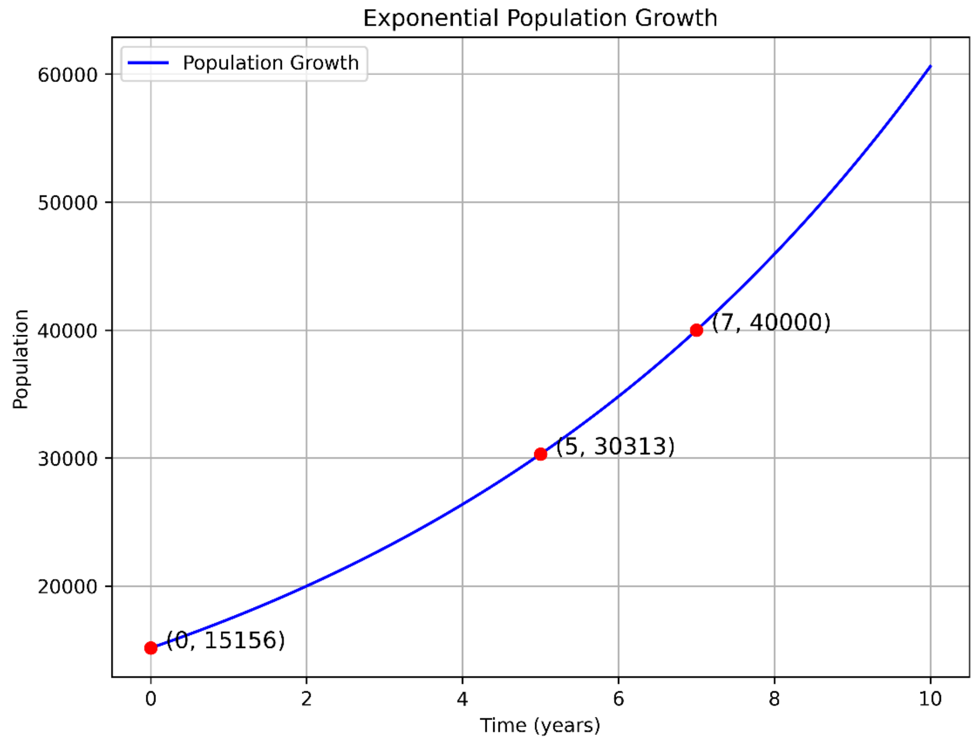

Fig. 1. Analytical solution of exponential population growth in a city.

Application 2: It is widely notable that the rate at which a radioactive material decays is related to its concentration. Determine the radioactive substance's half-life if there are 100 milligrams of it originally and it is seen that after five hours, the substance has lost 25% of its initial mass.

Solutions: This issue can be expressed mathematically as:

$$\frac{dy(t)}{dt} = -ky(t) \qquad (15)$$

where y is the radioactive material amount at time t and k is the proportionality constant. Regarding y_0 as the initial radioactive substance amount at $t = 0$.

Now employing the property 1, Rishi transform of derivative of function, on (15), we have

So we obtain $\frac{\varepsilon}{\sigma}Y(\varepsilon,\sigma) - \sum_{k=0}^{m-1}\left(\frac{\varepsilon}{\sigma}\right)^{k-1} y^{(m-1-k)}(0) = -k\mathcal{R}\{y(t)\}$

$$\frac{\varepsilon}{\sigma}Y(\varepsilon,\sigma) - \frac{\sigma}{\varepsilon}y(0) = -kY(\varepsilon,\sigma) \qquad (16)$$

Using initial condition $y(t_0) = y_0$ in (16) and on simplification, we have

$$\left(\frac{\varepsilon}{\sigma}+k\right)Y(\varepsilon,\sigma)=\frac{\sigma}{\varepsilon}y_0\,,\Rightarrow(\varepsilon+k\sigma)Y(\varepsilon,\sigma)=\frac{\sigma^2y_0}{\varepsilon}\Rightarrow Y(\varepsilon,\sigma)=\frac{\sigma^2y_0}{\varepsilon(\varepsilon+k\sigma)} \qquad (17)$$

Operating inverse Rishi transformation on each side of (17), we obtain

$$\mathcal{R}^{-1}\{Y(\varepsilon,\sigma)\}=\mathcal{R}^{-1}\left\{\frac{\sigma^2y_0}{\varepsilon(\varepsilon+k\sigma)}\right\}\Rightarrow y(t)=\mathcal{R}^{-1}\left\{\frac{\sigma^2y_0}{\varepsilon(\varepsilon+k\sigma)}\right\}$$

$$y(t)=y_0\mathcal{R}^{-1}\left\{\frac{\sigma^2}{\varepsilon(\varepsilon+k\sigma)}\right\}\Rightarrow y(t)=y_0e^{-kt} \qquad (18)$$

Now at $t=5$, the radioactive material has lost 25% of its initial mass 100 mg, thus $y=100-25=75$, making use of this in (18), we obtain

$$75=100e^{-5k}\Rightarrow e^{-5k}=0.75$$

$$\Rightarrow k=-0.2\,ln\,0.75=0.057 \qquad (19)$$

We required t when $y=\frac{y_0}{2}=\frac{100}{2}=50$ so from (18), we obtain

$$50=100e^{-kt} \qquad (20)$$

Replacing the value of k from (19) in (20), we obtain

$$50=100e^{-0.057t}$$

$$\Rightarrow e^{-0.057t}=0.5$$

$$\Rightarrow t=-\frac{1}{0.057}ln0.5$$

$$\Rightarrow t=12.16\text{ hours}$$

That is the necessary half-life of the radioactive substance.

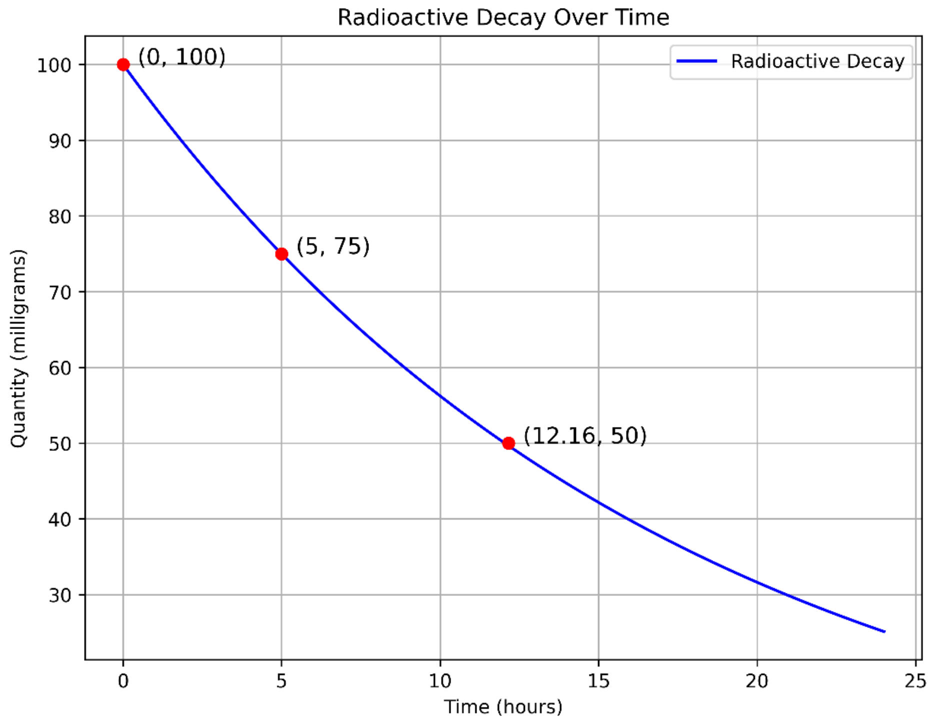

Fig. 2. Analytical solution of a radioactive material decay over time.

5. Conclusions

Rishi transform was effectively implemented to address problems related to population growth and decay. The applications presented in this study demonstrate the transform's capability to handle these issues with precision. Specifically, the results obtained from the Rishi transform were compared against established models such as the exponential growth and decay model. The comparison showed a high degree of accuracy, with the Rishi transform yielding results with least possible error compared to traditional integral transforms. This evidence supports the efficacy of the Rishi transform, suggesting its potential for future applications in various fields such as science, technology, and medicine through the establishment of mathematical models.

References

[1] S. A. Ahmad, S. K. Rafiq, H. D. M. Hilmi, H. U. Ahmed, Mathematical modeling techniques to predict the compressive strength of pervious concrete modified with waste glass powders, Asian J. Civ. Eng, 25(2024)773-785, https://doi.org/10.1007/s42107-023-00811-1

[2] B. M. Faraj, S. K. Rahman, D. A. Mohammed, H. D. Hilmi, A. Akgul, Efficient Finite Difference Approaches for Solving Initial Boundary Value Problems in Helmholtz Partial

Differential Equations, Contemp. Math, 4(2023)569-580, https://doi.org/10.37256/cm.4320232735

[3] H. Hilmi, S. J. MohammedFaeq, S. S. Fatah, Exact and Approximate Solution of Multi-Higher Order Fractional Differential Equations Via Sawi Transform and Sequential Approximation Method, J. Univ. Babylon Pure Appl. Sci., 32(2024)311-334, https://doi.org/10.29196/jtxxvm65

[4] S. Aggarwal, M. Pandey, N.Asthana, D.P Singh,N. Kumar, Application of Mahgoub Transform for Solving Population Growth and Decay Problems, J Comp. Math. Sci, 9(2018)1490-1496. https://doi.org/10.29055/jcms/892

[5] R. Saadeh, A.K Sedeeg, B. Ghazal, G. Gharih, Double Formable Integral Transform for Solving Heat Equations, Symmetry, 15(2023)1-15 https://doi.org/10.3390/sym15010218

[6] N. Güngör, Application of Kharrat-Toma Transform for Solving Linear Volterra Integral Equations, J Univ. Math, 3(2020)137-145. https://doi.org/10.33773/jum.829176

[7] R. Kumar, J. Chandel, S. Aggarwal, A new integral transform "Rishi Transform" with application, J. Sci. Res., 14(2022)521-532, http://dx.doi.org/10.3329/jsr.v14i2.54735

[8] A. Turab, H. Hilmi, J. L. Guirao, S. Jalil, N. Chorfi, P. O. Mohammed, The Rishi Transform method for solving multi-high order fractional differential equations with constant coefficients, AIMS Math 9(2024)3798-3809, https://doi.org/10.3934/math.2024187

[9] S. Aggarwal, R. Kumar, and J. Chandel, Exact solution of non-linear Volterra integral equation of first kind using Rishi transform, Bull. Pure & Applied Sci. Math. Stat 41(2022)159-166, https://doi.org/10.5958/2320-3226.2022.00022.4.

[10] S. Aggarwal, R. Kumar, and J. Chandel, Solution of Linear Volterra Integral Equation of Second Kind via Rishi Transform, J. Sci. Res. 15(2023), https://doi.org/10.3329/jsr.v15i1.60337

[11] W. S. Weiglhofer and K. A. Lindsay, Ordinary Differential Equations and Applications: Mathematical Methods for Applied Mathematicians, Physicists, Engineers and Bioscientists. Elsevier, (1999), https://doi.org/10.2307/3621558

[12] Z. Ahsan, The Potential of Fields in Einstein's Theory of Gravitation. Springer, (2019), https://doi.org/10.1007/978-981-13-8976-4

[13] C. Roberts, Ordinary differential equations: applications, models, and computing. CRC Press, (2011), https://doi.org/10.1201/b15111

[14] M. Braun, M. Golubitsky, Differential equations and their applications. Springer, (1983), https://doi.org/10.1007/978-1-4684-0164-6_3

[15] M. L. Abell and J. P. Braselton, Introductory differential equations. Academic Press, (2018), https://doi.org/10.1016/b978-0-12-814948-5.00006-9

[16] M. T. Hoang, M. Ehrhardt, Differential equation models for infectious diseases: Mathematical modeling, qualitative analysis, numerical methods and applications, (2024), https://doi.org/10.13140/RG.2.2.36191.59044

[17] G. C. Gorain, Introductory Course on Differential Equations, Alpha science international, (2014), https://doi.org/10.1016/c2012-0-07541-2

ريشي يتحول لحل مشاكل النمو السكاني والاضمحلال

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المستخلص

تهدف هذه الدراسة إلى استكشاف تطبيق تحويل ريشي على مشاكل النمو والاضمحلال السكاني، والتي تعد محورية في مجالات مختلفة مثل التمويل والعلوم الفيزيائية والكيمياء والأحياء وعلم الاجتماع. وتتمثل الأهداف الأساسية في إثبات فعالية تحويل ريشي في حل هذه المشاكل وتوفير العديد من التطبيقات العملية التي تبرز قيمتها وفائدتها. وتؤكد النتائج قدرة تحويل ريشي وفائدته في معالجة قضايا النمو والاضمحلال السكاني.