Decomposition matrices for the spin characters of S_{26} Modulo 11

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Abstract

We found in this paper the 11-decomposition matrices for the spin characters of S_{26}, which are a relationship between modular and ordinary characters

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1. Introduction

For any group there are three kinds of characters ordinary, modular (for a given prime p), and projective (for S_n called spin). The decomposition matrix is the relation between the ordinary and modular characters for a given prime p (see[1], [2]). Spin characters of S_n can be written as a liner combination with non-negative coefficients of the irreducible spin characters [2]. The case in $n = 25$ found by M. M. Jawad [3]. In this paper we use the techniques as given in [4]. The matrix for this spin from degree (247,207)[2], [5]. There are 50 blocks, the blocks B_1, B_2 are of defect two, the blocks $B_3,\dots, B_{17}$ are of defect one the others of defect zero. The notations used in this section can be found in [6].

Lemma (1). Brauer trees of blocks $B_3, B_4, \dots, B_{17}$ [7] are:

⟨24,2⟩*__⟨13,11,2⟩ = ⟨13,11,2⟩′__⟨13,10,2,1⟩*__⟨13,8,3,2⟩*__⟨13,7,4,2⟩*__⟨13,6,5,2⟩*,

⟨23,2,1⟩__⟨13,12,1⟩ \ ⟨12,11,2,1⟩* / ⟨12,8,3,2,1⟩__⟨12,7,4,2,1⟩__⟨12,6,5,2,1⟩
⟨23,2,1⟩′__⟨13,12,1⟩′ / ⟨12,11,2,1⟩* \ ⟨12,8,3,2,1⟩′__⟨12,7,4,2,1⟩′__⟨12,6,5,2,1⟩′ ,

⟨25,5⟩*__⟨16,10⟩*__⟨11,10,5⟩ = ⟨11,10,5⟩′__⟨10,9,5,2⟩*__⟨10,8,5,2⟩*__⟨10, ,7,5,4⟩*,

⟨21,3,2⟩__⟨14,10,2⟩__⟨13,10,3⟩ \ ⟨11,10,3,2⟩* / ⟨10,7,4,3,2⟩__⟨10,6,5,3,2⟩
⟨21,3,2⟩′__⟨14,10,2⟩′__⟨13,10,3⟩′ / ⟨11,10,3,2⟩* \ ⟨10,7,4,3,2⟩′__⟨10,6,5,3,2⟩′ ,

⟨20,6⟩*__⟨17,9⟩*__⟨11,9,6⟩ = ⟨11,9,6⟩′__⟨10,9,6,1⟩*__⟨9,8,6,3⟩*__⟨9,7,6,4⟩*,

⟨20,5,1⟩__⟨16,9,1⟩__⟨12,9,5⟩ \ ⟨11,9,5,1⟩* / ⟨9,8,5,3,1⟩__⟨9,7,5,4,1⟩
⟨20,5,1⟩′__⟨16,9,1⟩′__⟨12,9,5⟩′ / ⟨11,9,5,1⟩* \ ⟨9,8,5,3,1⟩′__⟨9,7,5,4,1⟩′ ,

$\langle 19,7\rangle^{*}\_\_\langle 18,8\rangle^{*}\_\_\langle 11,8,7\rangle = \langle 11,8,7\rangle'\_\_\langle 10,8,7,1\rangle^{*}\_\_\langle 9,8,7,2\rangle^{*}\_\_\langle 8,7,6,5\rangle^{*}$,

$\langle 19,6,1\rangle\_\_\langle 17,8,1\rangle\_\_\langle 12,8,6\rangle$ \ $\langle 11,8,6,1\rangle^{*}$ / $\langle 9,8,6,2,1\rangle\_\_\langle 8,7,6,4,1\rangle$
$\langle 19,6,1\rangle'\_\_\langle 17,8,1\rangle'\_\_\langle 12,8,6\rangle'$ / $\langle 11,8,6,1\rangle^{*}$ \ $\langle 9,8,6,2,1\rangle'\_\_\langle 8,7,6,4,1\rangle'$,

$\langle 19,5,2\rangle\_\_\langle 16,8,2\rangle\_\_\langle 13,8,5\rangle$ \ $\langle 11,8,5,2\rangle^{*}$ / $\langle 10,8,5,2,1\rangle\_\_\langle 8,7,5,4,2\rangle$
$\langle 19,5,2\rangle'\_\_\langle 16,8,2\rangle'\_\_\langle 13,8,5\rangle'$ / $\langle 11,8,5,2\rangle^{*}$ \ $\langle 10,8,5,2,1\rangle'\_\_\langle 8,7,5,4,2\rangle'$,

$\langle 19,4,2,1\rangle^{*}\_\_\langle 15,8,2,1\rangle^{*}\_\_\langle 13,8,4,1\rangle^{*}\_\_\langle 12,8,4,2\rangle^{*}\_\_\langle 11,8,4,2,1\rangle =$
$\langle 11,8,4,2,1\rangle'\_\_\langle 8,6,5,4,2\rangle^{*}$,

$\langle 18,6,2\rangle\_\_\langle 17,7,2\rangle\_\_\langle 13,7,6\rangle$ \ $\langle 11,7,6,2\rangle^{*}$ / $\langle 10,7,6,2,1\rangle\_\_\langle 8,7,6,3,2\rangle$
$\langle 18,6,2\rangle'\_\_\langle 17,7,2\rangle'\_\_\langle 13,7,6\rangle'$ / $\langle 11,7,6,2\rangle^{*}$ \ $\langle 10,7,6,2,1\rangle'\_\_\langle 8,7,6,3,2\rangle'$,

$\langle 18,5,3\rangle\_\_\langle 16,7,3\rangle\_\_\langle 14,7,5\rangle$ \ $\langle 11,7,5,3\rangle^{*}$ / $\langle 10,7,5,3,1\rangle\_\_\langle 9,7,5,3,2\rangle$
$\langle 18,5,3\rangle'\_\_\langle 16,7,3\rangle'\_\_\langle 14,7,5\rangle'$ / $\langle 11,7,5,3\rangle^{*}$ \ $\langle 10,7,5,3,1\rangle'\_\_\langle 9,7,5,3,2\rangle'$,

$\langle 18,5,2,1\rangle^{*}\_\_\langle 16,7,2,1\rangle^{*}\_\_\langle 13,7,5,1\rangle^{*}\_\_\langle 12,7,5,2\rangle^{*}\_\_\langle 11,7,5,2,1\rangle =$
$\langle 11,7,5,2,1\rangle'\_\_\langle 8,7,5,3,2,1\rangle^{*}$,

$\langle 17,4,3,2\rangle^{*}\_\_\langle 15,6,3,2\rangle^{*}\_\_\langle 14,6,4,2\rangle^{*}\_\_\langle 13,6,4,3\rangle^{*}\_\_\langle 11,6,4,3,2\rangle =$
$\langle 11,6,4,3,2\rangle'\_\_\langle 10,6,4,3,2,1\rangle^{*}$,

$\langle 16,4,3,2,1\rangle\_\_\langle 15,5,3,2,1\rangle\_\_\langle 14,5,4,2,1\rangle\_\_\langle 13,5,4,3,1\rangle\_\_\langle 12,5,4,3,2\rangle$ \ $\langle 11,5,4,3,2,1\rangle^{*}$
$\langle 16,4,3,2,1\rangle'\_\_\langle 15,5,3,2,1\rangle'\_\_\langle 14,5,4,2,1\rangle'\_\_\langle 13,5,4,3,1\rangle'\_\_\langle 12,5,4,3,2\rangle'$ / $\langle 11,5,4,3,2,1\rangle^{*}$
respectively.

Proof. For B_4, we use the $(r,\bar{r})$-inducing p.i.s. of D_{25}, D_{41}, D_{43}, D_{44},…, D_{48}, of S_{25} to S_{26} we get k_1, k_2, d_{70}, $d_{71}, \dots d_{75}$, and since it's associate(see [2]), so $\langle 23,2,1\rangle \neq \langle 23,2,1\rangle'$ it follows that k_1 splits or there are two columns:
$Y_1 = a_1\langle 23,2,1\rangle + a_2\langle 13,12,1\rangle + a_3\langle 12,11,2,1\rangle^{*} + a_4\langle 12,8,3,2,1\rangle + a_5\langle 12,7,4,2,1\rangle + a_6\langle 12,6,5,2,1\rangle$,
$Y_2 = a_1\langle 23,2,1\rangle' + a_2\langle 13,12,1\rangle' + a_3\langle 12,11,2,1\rangle^{*} + a_4\langle 12,8,3,2,1\rangle' + a_5\langle 12,7,4,2,1\rangle' + a_6\langle 12,6,5,2,1\rangle'$,
so that $a_1, a_2, \dots, a_6 \in \{0,1\}$[7]. Let $a_1 = 1$. Since $\langle 23,2,1\rangle \downarrow S_{25} \cap \langle 12,8,3,2,1\rangle \downarrow S_{25}$ has no i.m.s, so $a_4 = 0$. The same way, we find $a_5, a_6 = 0$. But $\deg Y_1, Y_2 \equiv 0 \bmod 11^2$ only when $a_1, a_2 = 1, a_3 = 0$. It follows that $k_1 = d_{66} + d_{67}$. Since B_4 of defect one. So $k_2 = d_{68} + d_{69}$ [3]. In the same way B_6, we use $(r,\bar{r})$-inducing and to get $k_1, k_2, \dots, k_5$. Since it's associate[5]. So k_1 divided or there are columns:
$Y_1 = a_1\langle 21,3,1\rangle + a_2\langle 14,10,2\rangle + a_3\langle 13,10,3\rangle + a_4\langle 11,10,3,2\rangle^{*} + a_5\langle 10,7,4,3,2\rangle + a_6\langle 10,6,5,3,2\rangle$,
$Y_2 = a_1\langle 21,3,1\rangle' + a_2\langle 14,10,2\rangle' + a_3\langle 13,10,3\rangle' + a_4\langle 11,10,3,2\rangle^{*} + a_5\langle 10,7,4,3,2\rangle' + a_6\langle 10,6,5,3,2\rangle'$, $a_1, a_2, \dots, a_6 \in \{0,1\}$. Let $a_1 = 1$, and $\langle 21,3,2\rangle \downarrow S_{25} \cap \langle 13,10,3,2\rangle \downarrow S_{25}$has

no i.m.s. So $a_3 = 0$. In the same way we get $a_4, a_5, a_6 = 0$. Then $\deg Y_1, Y_2 \equiv 0 \bmod 11^2$ only when $a_1, a_2 = 1, a_3 = 0$ so $k_1 = d_{81} + d_{82}$.In the same we divided k_2, k_5 to d_{83}, d_{84}, d_{89}, d_{90} respectively. But B_6 of defect one so k_3, k_4 splits to $d_{85}, d_{86}, d_{87}, d_{88}$. In the way we prove the blocks B_{11}, B_{13}, B_{14}. For B_8, we use the $(r, \bar{r})$-inducing to get on $d_{96}, d_{97}, k_1, k_2, d_{102}, \ldots, d_{105}$. Since it's associate so k_1 divided or there Y_1, Y_2 such that:
$Y_1 = a_1\langle 16,9,1\rangle + a_2\langle 12,9,5\rangle + a_3\langle 11,9,5,1\rangle^* + a_4\langle 9,8,5,3,1\rangle + a_5\langle 9,7,5,4,1\rangle$,
$Y_2 = a_1\langle 16,9,1\rangle' + a_2\langle 12,9,5\rangle' + a_3\langle 11,9,5,1\rangle^* + a_4\langle 9,8,5,3,1\rangle' + a_5\langle 9,7,5,4,1\rangle'$,
$a_1, a_2, \ldots, a_5 \in \{0,1\}$. In the same above $\deg Y_1, Y_2 \equiv 0 \bmod 11^2$ only when $a_1, a_2 = 1$ so $k_1 = d_{96} + d_{97}$. But it's of defect one so $k_2 = d_{100} + d_{101}$. In the same way mentional above we discuss the block B_{10} and B_{17}. The other we found it by use the $(r, \bar{r})$-inducing from S_{25} to S_{26}directly. Finally we find the Brauer trees for the blocks $B_3, B_4, \ldots, B_{17}$ of the decomposition that we found [7].

Lemma (2). Decomposition matrix for the block B_2 of a double [7] is a table (1).

$\langle 25,1\rangle^*$	1																			
$\langle 23,3\rangle^*$	1	1																		
$\langle 22,3,1\rangle$		1	1																	
$\langle 20,3,2,1\rangle^*$			1	1																
$\langle 18,4,3,1\rangle^*$				1	1															
$\langle 17,5,3,1\rangle^*$					1	1														
$\langle 16,6,3,1\rangle^*$					1	1	1													
$\langle 15,7,3,1\rangle^*$				1	1		1	1												
$\langle 14,12\rangle^*$		1							1											
$\langle 14,11,1\rangle$	1	1	1						1	1										
$\langle 14,9,2,1\rangle^*$			1	1						1	1									
$\langle 14,8,3,1\rangle^*$				1				1			1	1								
$\langle 14,7,4,1\rangle^*$							1	1				1	1							
$\langle 14,6,5,1\rangle^*$							1						1							
$\langle 13,9,3,1\rangle^*$										1	1	1		1						
$\langle 12,11,3\rangle$	1								1	1					1					
$\langle 12,10,3,1\rangle^*$									2	1				1	2	2				
$\langle 12,9,3,2\rangle^*$												1		1	1	2	1			
$\langle 12,7,4,3\rangle^*$												1	1				1	1		
$\langle 12,6,5,3\rangle^*$													1					1		
$\langle 11,9,3,2,1\rangle$															1	1	1		1	
$\langle 11,7,4,3,1\rangle$																1	1	1	1	1
$\langle 11,6,5,3,1\rangle$																		1		1
$\langle 9,7,4,3,2,1\rangle^*$																1				1
$\langle 9,6,5,3,2,1\rangle^*$																			1	1
$\langle 7,6,5,4,3,1\rangle^*$																			1	
	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}	d_{57}	d_{58}	d_{59}	d_{60}

table (1)

Proof. Using $(r, \bar{r})$ $-$inducing of p.i.s. $D_{21}, D_{23}, D_3, D_4, \ldots, D_8, D_{39}, D_{10}, D_{11}, D_{45}, D_{47}, D_{158}$, $D_{39}, D_{16}, D_{53}, D_{55}, D_{19}, D_{20}$ of S_{25} to S_{26}, we got $d_{41}, d_{42}, \ldots, d_{60}$ respectively. Since $(d_{45} - d_{46}) \downarrow_{(1,0)} S_{25}$ is not p.s., so $d_{46} \not\subset d_{45}$, consequently we get the table (1).

Theorem (3). Decomposition matrix for spin character of S_{26} the block B_1 is table (2).

$\langle 26\rangle$	1																																							
$\langle 26\rangle'$		1																																						
$\langle 22,4\rangle^*$	1	1	1	1																																				
$\langle 21,4,1\rangle$			1		1																																			
$\langle 22,4,1\rangle'$				1		1																																		
$\langle 20,4,2\rangle$					1		1																																	
$\langle 20,4,2\rangle'$						1		1																																
$\langle 19,4,3\rangle$							1		1																															
$\langle 19,4,3\rangle'$								1		1																														
$\langle 17,5,4\rangle$									1		1																													
$\langle 17,5,4\rangle'$										1		1																												
$\langle 16,6,4\rangle$									1		1		1																											
$\langle 16,6,4\rangle'$										1		1		1																										
$\langle 15,11\rangle^*$	1	1	1	1											1	1																								
$\langle 15,10,1\rangle$	1	1	1		1										1		1																							
$\langle 15,10,1\rangle'$	1	1		1		1										1		1																						
$\langle 15,9,2\rangle$					1		1										1		1																					
$\langle 15,9,2\rangle'$						1		1										1		1																				
$\langle 15,8,3\rangle$							1		1										1		1																			
$\langle 15,8,3\rangle'$								1		1										1		1																		
$\langle 15,7,4\rangle$									1				1								1		1																	
$\langle 15,7,4\rangle'$										1				1								1		1																
$\langle 15,6,5\rangle$													1										1																	
$\langle 15,6,5\rangle'$														1										1																
$\langle 14,8,4\rangle$																			1		1		1		1															
$\langle 14,8,4\rangle'$																				1		1		1		1														
$\langle 13,9,4\rangle$																	1		1						1		1													
$\langle 13,9,4\rangle'$																		1		1						1		1												
$\langle 12,10,4\rangle$	1	1													1	1	1										1		1	1										
$\langle 12,10,4\rangle'$	1	1													1	1		1										1	1	1										
$\langle 11,10,4,1\rangle^*$															1	1											1	1	1	1	1	1								
$\langle 11,9,4,2\rangle^*$																									1	1	1	1	1	1	1	1	1	1						
$\langle 11,8,4,3\rangle^*$																							1	1	1	1							1	1	1	1				
$\langle 11,6,5,4\rangle^*$																							1	1											1	1				
$\langle 10,9,4,2,1\rangle$																													1				1				1			
$\langle 10,9,4,2,1\rangle'$																														1				1				1		
$\langle 10,8,4,3,1\rangle$																															1		1		1		1		1	
$\langle 10,8,4,3,1\rangle'$																																1		1		1		1		1
$\langle 10,6,5,4,1\rangle$																																			1				1	
$\langle 10,6,5,4,1\rangle'$																																				1				1
$\langle 9,8,4,3,2\rangle$																															1								1	
$\langle 9,8,4,3,2\rangle'$																																1								1
$\langle 9,6,5,4,2\rangle$																																					1		1	
$\langle 9,6,5,4,2\rangle'$																																						1		1
$\langle 8,6,5,4,3\rangle$																																					1			
$\langle 8,6,5,4,3\rangle'$																																						1		
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

table (2)

Proof. By using $(r,\bar{r})$-inducing of S_{25} to S_{26}, we get approximation matrix in table (3).

$\langle 26\rangle$	1																									
$\langle 26\rangle'$	1																									
$\langle 22,4\rangle^*$	2	1	1																							
$\langle 21,4,1\rangle$		1		1																					a_1	
$\langle 22,4,1\rangle'$			1	1																						a_1
$\langle 20,4,2\rangle$				1	1																				a_2	
$\langle 20,4,2\rangle'$				1	1																					a_2
$\langle 19,4,3\rangle$					1	1																			a_3	
$\langle 19,4,3\rangle'$					1	1																				a_3
$\langle 17,5,4\rangle$						1	1																		a_4	
$\langle 17,5,4\rangle'$						1	1																			a_4
$\langle 16,6,4\rangle$						1	1	1																	a_5	
$\langle 16,6,4\rangle'$						1	1	1																		a_5
$\langle 15,11\rangle^*$	2	1	1						1	1															a_6	a_6
$\langle 15,10,1\rangle$	2	1		1					1		1														a_7	
$\langle 15,10,1\rangle'$	2		1	1						1	1															a_7
$\langle 15,9,2\rangle$				1	1						1	1													a_8	
$\langle 15,9,2\rangle'$				1	1						1	1														a_8
$\langle 15,8,3\rangle$					1	1		1				1	1												a_9	
$\langle 15,8,3\rangle'$					1	1		1				1	1													a_9
$\langle 15,7,4\rangle$						1		2					1	1											a_{10}	
$\langle 15,7,4\rangle'$						1		2					1	1												a_{10}
$\langle 15,6,5\rangle$								1						1											a_{11}	
$\langle 15,6,5\rangle'$								1						1												a_{11}
$\langle 14,8,4\rangle$								1				1	1	1	1										a_{12}	
$\langle 14,8,4\rangle'$								1				1	1	1	1											a_{12}
$\langle 13,9,4\rangle$											1	1			1	1									a_{13}	
$\langle 13,9,4\rangle'$											1	1			1	1										a_{13}
$\langle 12,10,4\rangle$	2								1	1	1					1	1	1							a_{14}	
$\langle 12,10,4\rangle'$	2								1	1	1					1	1	1								a_{14}
$\langle 11,10,4,1\rangle^*$									1	1						2	1	1	2						a_{15}	a_{15}
$\langle 11,9,4,2\rangle^*$															2	2	1	1	2	1	1				a_{16}	a_{16}
$\langle 11,8,4,3\rangle^*$														2	2					1	1	1	1		a_{17}	a_{17}
$\langle 11,6,5,4\rangle^*$														2								1	1		a_{18}	a_{18}
$\langle 10,9,4,2,1\rangle$																	1			1				1	a_{19}	
$\langle 10,9,4,2,1\rangle'$																		1			1			1		a_{19}
$\langle 10,8,4,3,1\rangle$																			1	1		1		1	1	a_{20}	
$\langle 10,8,4,3,1\rangle'$																			1		1		1	1	1		a_{20}
$\langle 10,6,5,4,1\rangle$																						1			1	a_{21}	
$\langle 10,6,5,4,1\rangle'$																							1		1		a_{21}
$\langle 9,8,4,3,2\rangle$																			1						1	a_{22}	
$\langle 9,8,4,3,2\rangle'$																			1						1		a_{22}
$\langle 9,6,5,4,2\rangle$																								1	1	a_{23}	
$\langle 9,6,5,4,2\rangle'$																								1	1		a_{23}
$\langle 8,6,5,4,3\rangle$																								1		a_{24}	
$\langle 8,6,5,4,3\rangle'$																								1			a_{24}
	k_1	d_3	d_4	k_2	k_3	k_4	k_5	c	d_{15}	d_{16}	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	d_{29}	d_{30}	k_{13}	d_{33}	d_{34}	d_{35}	d_{36}	k_{14}	k_{15}	Y_1	Y_2

table(3)

Since$(k_4 - k_5) \downarrow_{(8,4)} S_{25}$ is not p.s., so $k_5 \not\subset k_4$. However $k_9 \subset c$ we prove that by the way of contradiction. Let $(\langle 14,8,4\rangle - \langle 15,6,5\rangle)$ is m.s. for S_{26} but $\langle 14,8,4\rangle - \langle 15,6,5\rangle \downarrow_{(8,4)} S_{25}$ is not m.s., so$\therefore$ $k_6 = c - k_9$.

From [3] k_1 must split to d_1,d_2. Also $\langle 20,4,2\rangle \neq \langle 20,4,2\rangle'$ on $(11,\alpha)$-regular classes (see[4]). So $k_2 = d_5 + d_6$ or there are two columns. If there are two columns Y_1, Y_2 then we have the approximation matrix as above, to describe it's such that $\langle 21,4,1\rangle \downarrow S_{25}$ has 4 of i.m.s. and, from the table(3) we get $a_1 \in \{0,1,2\}$, in the same way we get $a_3, a_4, a_{11}, a_{18}, a_{24} \in \{0,1\}$,$a_2, a_5, a_6, a_{21}, a_{22}, a_{23} \in \{0,1,2\}$,$a_7, a_8, a_{15}, a_{17}, a_{19} \in \{0,1,\dots,4\}$,$a_{10}, a_{13} \in \{0,1,\dots,5\}$, $a_9, a_{12}, a_{14}, a_{16}, a_{20} \in \{0,1,\dots,6\}$. Take $a_2 \in \{1,2\}$ (if $a_2 = 0$ then we have contradiction) and, since $\langle 20,4,2\rangle \downarrow S_{25} \cap \langle 17,5,4\rangle \downarrow S_{25}$ has no i.m.s, so we have $a_4 = 0$ by counting the intersections we get on $a_5, a_6, a_{11}, a_{13}, a_{14}, \dots, a_{24}$ are equal to zero, and since inducing m.s. is m.s. [8] we have:

$(\langle 21,4\rangle - \langle 15,10\rangle + \langle 11,10,4\rangle^*) \uparrow^{(1,11)} S_{26} \Rightarrow a_1 \geq a_7$ (1)

$(\langle 15,10\rangle - \langle 21,4\rangle) \uparrow^{(1,11)} S_{26} \Rightarrow a_7 \geq a_1$. Hence $a_1 = a_7$ (2)

$(\langle 20,4,1\rangle^* - \langle 15,9,1\rangle + \langle 12,9,4\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_2 \geq a_8$ (3)

$(\langle 15,9,1\rangle^* - \langle 20,4,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_8 \geq a_2$. Hence $a_2 = a_8$ (4)

$(\langle 14,8,3\rangle^* - \langle 15,7,3\rangle^* + \langle 16,6,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{12} \geq a_{10}$ (5)

$(\langle 15,7,3\rangle^* - \langle 14,8,3\rangle^* + \langle 13,9,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} \geq a_{12}$. Hence $a_{10} = a_{12}$ (6)

$(\langle 20,3,2\rangle^* - \langle 21,3,1\rangle^* - \langle 18,4,3\rangle^* + \langle 22,3\rangle + \langle 17,5,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_2 \geq a_1 + a_3$ (7)

$(\langle 18,4,3\rangle + \langle 21,3,1\rangle^* - \langle 20,3,2\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_3 + a_1 \geq a_2$. Hence$a_2 = a_1 + a_3$ (8)

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = n_1 k_2 + n_2 k_3 + n_3 k_9$, $n_2 = 0, n_1 \in \{1,2\}$, $n_3 \in \{0,1,\dots,5\}$ or $n_2 = 1, n_1 \in \{0,1\}, n_3 \in \{0,1,\dots,5\}$. So $k_2 = d_5 + d_6$.
Also $\langle 19,4,3\rangle \neq \langle 19,4,3\rangle'$ on $(11,\alpha)$ −regular classes . It follows that k_3 or k_4 splits or there are Y_1, Y_2 . Let $a_3 = 1$, and use the same way above, we get:
$Y_1 = a_2\langle 20,4,2\rangle + \langle 19,4,3\rangle + a_8\langle 15,9,2\rangle + a_9\langle 15,8,3\rangle + a_{10}\langle 15,7,4\rangle + a_{12}\langle 14,8,4\rangle$,
$Y_2 = a_2\langle 20,4,2\rangle' + \langle 19,4,3\rangle' + a_8\langle 15,9,2\rangle' + a_9\langle 15,8,3\rangle' + a_{10}\langle 15,7,4\rangle' + a_{12}\langle 14,8,4\rangle'$
Since

$(\langle 20,4,1\rangle^* - \langle 15,9,1\rangle^* + \langle 12,9,4\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_2 \geq a_8$ (9)

$(\langle 15,9,1\rangle^* - \langle 20,4,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_8 \geq a_2$. Hence $a_2 = a_8$ (10)

$(\langle 15,7,3\rangle^* - \langle 14,8,3\rangle^* + \langle 13,9,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} \geq a_{12}$ (11)

$(\langle 14,8,3\rangle^* - \langle 15,7,3\rangle^* + \langle 16,6,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{12} \geq a_{10}$. Hence $a_{10} = a_{12}$ (12)

$(\langle 15,8,2\rangle - \langle 19,4,2\rangle - \langle 13,8,4\rangle + \langle 11,8,4,2\rangle) \uparrow^{(9,3)} S_{26} \Rightarrow a_9 \geq a_3 + a_{10}$ (13)

$(\langle 19,4,2\rangle + \langle 13,8,4\rangle - \langle 15,8,2\rangle) \uparrow^{(9,3)} S_{26} \Rightarrow a_3 + a_{10} \geq a_9$.
Hence $a_9 = a_3 + a_{10}$ (14)

$(\langle 14,8,3\rangle^* - \langle 14,9,2\rangle^* + \langle 14,10,1\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} + a_9 \geq a_2$ (15)

$(\langle 14,9,2\rangle^* - \langle 14,8,3\rangle^* + \langle 13,9,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_2 \geq a_9 + a_{10}$.
Hence $a_2 = a_9 + a_{10}$, (16)

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $a_2, a_3, a_8, a_9 = 1, a_{10}, a_{12} = 0$. So $k_3 = d_7 + d_8$.

Also $\langle 17,5,4\rangle \neq \langle 17,5,4\rangle'$ on $(11,\alpha)$ −regular classes. Suppose that $a_4 = 1$. Using the same technic then we get deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $a_4, a_5 = 1$. So k_4, k_5 are split to $d_9, d_{10}, d_{11}, d_{12}$ respectively.

For k_6, we have $\langle 16,6,4\rangle \neq \langle 16,6,4\rangle'$. So it's splits or there are other Y_1, Y_2. So let $a_5 \in \{1,2\}$. By restricting and inducing, we get $Y_1 = a_5\langle 16,6,4\rangle + a_9\langle 15,8,3\rangle + a_{10}\langle 15,7,4\rangle + a_{11}\langle 15,6,5\rangle + a_{12}\langle 14,8,4\rangle$, $Y_2 = a_5\langle 16,6,4\rangle' + a_9\langle 15,8,3\rangle' + a_{10}\langle 15,7,4\rangle' + a_{11}\langle 15,6,5\rangle' + a_{12}\langle 14,8,4\rangle'$ and since

$(\langle 13,8,4\rangle^* + \langle 19,4,2\rangle^* - \langle 15,8,2\rangle^*) \uparrow^{(9,3)} S_{26} \Rightarrow a_{12} \geq a_9$ (17)

$(\langle 15,8,2\rangle^* - \langle 13,8,4\rangle^* + \langle 11,8,4,2\rangle^*) \uparrow^{(9,3)} S_{26} \Rightarrow a_9 \geq a_{12}$. Hence $a_9 = a_{12}$, (18)

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = k_6 + nk_9, n \in \{0,1,\ldots,4\}$. So $k_6 = d_{13} + d_{14}$.

For k_8, we have $\langle 15,9,2\rangle \neq \langle 15,9,2\rangle'$. Let $a_8 \in \{1,\ldots,4\}$. By restricting and inducing, we get: $Y_1 = a_8\langle 15,9,2\rangle + a_9\langle 15,8,3\rangle + a_{10}\langle 15,7,4\rangle + a_{12}\langle 14,8,4\rangle + a_{13}\langle 13,9,4\rangle + a_{14}\langle 12,10,4\rangle$, $Y_2 = a_8\langle 15,9,2\rangle' + a_9\langle 15,8,3\rangle' + a_{10}\langle 15,7,4\rangle' + a_{12}\langle 14,8,4\rangle' + a_{13}\langle 13,9,4\rangle' + a_{14}\langle 12,10,4\rangle'$ and, since

$(\langle 15,10\rangle + \langle 15,10\rangle' + \langle 10,9,4,2\rangle + \langle 10,9,4,2\rangle' - \langle 11,10,4\rangle^*) \uparrow^{(1,11)} S_{26} \Rightarrow 0 \geq a_{14}$. Hence $a_{14} = 0$ (19)

$(\langle 15,9,1\rangle^* - \langle 12,9,4\rangle^* + \langle 11,9,4,1\rangle) \uparrow^{(10,2)} S_{26} \Rightarrow a_8 \geq a_{13}$ (20)

$(\langle 12,9,4\rangle^* - \langle 15,9,1\rangle^* + \langle 20,4,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{13} \geq a_8$. Hence $a_8 = a_{13}$ (21)

$(\langle 15,8,2\rangle^* - \langle 13,8,4\rangle^* + \langle 11,8,4,2\rangle) \uparrow^{(9,3)} S_{26} \Rightarrow a_9 \geq a_{12}$ (22)

$(\langle 13,8,4\rangle - \langle 15,8,2\rangle^* + \langle 19,4,2\rangle^*) \uparrow^{(9,3)} S_{26} \Rightarrow a_{12} \geq a_9$,. Hence $a_9 = a_{12}$, (23)

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = n_1k_8 + n_2k_9$, such that $n_1 \in \{1,2,3,4\}$, $n_2 \in \{0,1,\ldots,6-n_1\}$, so $k_8 = d_{19} + d_{20}$.

Also $\langle 15,8,3\rangle \neq \langle 15,8,3\rangle'$. Let $a_9 \in \{1,\ldots,6\}$. By restricting and inducing we get: $Y_1 = a_9\langle 15,8,3\rangle + a_{10}\langle 15,7,4\rangle + a_{11}\langle 15,6,5\rangle + a_{12}\langle 14,8,4\rangle + a_{13}\langle 13,9,4\rangle + a_{16}\langle 11,9,4,2\rangle^* + a_{17}\langle 11,8,4,3\rangle^*$, $Y_2 = a_9\langle 15,8,3\rangle' + a_{10}\langle 15,7,4\rangle' + a_{11}\langle 15,6,5\rangle' + a_{12}\langle 14,8,4\rangle' + a_{13}\langle 13,9,4\rangle' + a_{16}\langle 11,9,4,2\rangle^* + a_{17}\langle 11,8,4,3\rangle^*$ and since

$(\langle 12,9,4\rangle^* - \langle 11,9,4,1\rangle + \langle 9,8,4,3,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{13} \geq a_{16}$ (24)

$(\langle 11,9,4,1\rangle - \langle 12,9,4\rangle^* + \langle 15,9,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{16} \geq a_{13}$. Hence $a_{13} = a_{16}$ (25)

$(\langle 16,6,3\rangle^* - \langle 14,6,5\rangle^* + \langle 11,6,5,3\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow 0 \geq a_{11}$. Hence $a_{11} = 0$ (26)

$(\langle 10,9,4,2\rangle - \langle 10,8,4,3\rangle + \langle 10,6,5,4\rangle) \uparrow^{(1,11)} S_{26} \Rightarrow a_{13} \geq a_{17}$ (27)

$(\langle 10,8,4,3\rangle - \langle 10,9,4,2\rangle^* + \langle 11,10,4\rangle^*) \uparrow^{(1,11)} S_{26} \Rightarrow a_{17} \geq a_{13}$.
Hence $a_{13} = a_{17}$ (28)

$(\langle 14,8,3\rangle^* + \langle 15,6,5\rangle^* - \langle 14,7,4\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_9 \geq a_{10}$ (29)

$(\langle 14,7,4\rangle^* + \langle 14,9,2\rangle^* - \langle 14,8,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{10} \geq a_9$. Hence $a_9 = a_{10}$, (30)

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = n_1k_9 + n_2k_{11}$, $n_1 = 1, n_2 \in \{0,1,\ldots,4\}$ or $n_1 \in \{2,3,\ldots,5\}$, $n_2 \in \{0,1,\ldots,6-n_1\}$. So $k_9 = d_{21} + d_{22}$.

Since $\langle 15,6,4\rangle \neq \langle 15,6,4\rangle'$, let $a_{11} = 1$. By restricting and inducing we get:

$Y_1 = a_{10}\langle 15,7,4\rangle + a_{11}\langle 15,6,5\rangle + +a_{17}\langle 11,8,4,3\rangle^* + a_{18}\langle 11,6,5,4\rangle^*$,$Y_2 = a_{10}\langle 15,7,4\rangle' + a_{11}\langle 15,6,5\rangle' + a_{12}\langle 14,8,4\rangle' + a_{17}\langle 11,8,4,3\rangle^* + a_{18}\langle 11,6,5,4\rangle^*$ and, since

$$(\langle 11,6,5,3\rangle - \langle 11,7,4,3\rangle + \langle 11,9,4,1\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{18} \geq a_{11} \qquad (31)$$

We set deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $a_{10} = a_{12} = a_{17} = a_{18} = 1$. It follows that $k_{10} = d_{23} + d_{24}$.

For k_{11} let $a_{12} \in \{1, \dots, 6\}$. By restricting and inducing we get on and, since

$$(\langle 13,9,3\rangle^* - \langle 14,8,3\rangle^* + \langle 15,7,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{13} \geq a_{12} \qquad (32)$$

$$(\langle 14,8,3\rangle^* + \langle 12,10,3\rangle^* - \langle 13,9,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{12} \geq a_{13}. \text{Hence } a_{12} = a_{13} \qquad (33)$$

$$(\langle 12,9,4\rangle^* - \langle 11,9,4,1\rangle + \langle 9,8,4,3,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{12} \geq a_{16} \qquad (34)$$

$$(\langle 11,9,4,1\rangle - \langle 12,9,4\rangle^* + \langle 15,9,1\rangle^*) \uparrow^{(10,2)} S_{26} \Rightarrow a_{16} \geq a_{12}. \text{Hence } a_{12} = a_{16}, \qquad (35)$$

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = nk_{11}, n \in \{1,2,3,4\}$, so $k_{11} = d_{25} + d_{26}$.

For k_{12} let $a_{13} \in \{1, \dots, 5\}$. By restricting and inducing, we get

$$(\langle 11,10,3,1\rangle - \langle 11,9,3,2\rangle + \langle 10,9,3,2,1\rangle^* + \langle 14,7,4\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{15} \geq a_{16} \qquad (36)$$

$$(\langle 11,9,3,2\rangle - \langle 11,10,3,1\rangle + \langle 14,11\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{16} \geq a_{15}. \text{Hence } a_{15} = a_{16} \qquad (37)$$

$$(\langle 11,10,3,1\rangle - \langle 13,9,3\rangle^* + \langle 14,8,3\rangle^* + \langle 14,9,2\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{15} \geq a_{13} \qquad (38)$$

$$(\langle 13,9,3\rangle^* - \langle 11,10,3,1\rangle + \langle 14,11\rangle + \langle 10,9,3,2,1\rangle^* + \langle 10,7,4,3,1\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{13} \geq a_{15}. \text{Hence } a_{13} = a_{15}. \qquad (39)$$

Then $k_{12} = d_{27} + d_{28}$. For k_{13} let $a_{14} \in \{1,2\}$. By restricting and inducing we get on:

$$(\langle 11,6,5,3\rangle - \langle 11,7,4,3\rangle + \langle 10,9,3,2,1\rangle^* + \langle 13,9,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{17} = 0 \qquad (40)$$

$$(\langle 11,10,3,1\rangle - \langle 11,6,3,2\rangle + \langle 11,7,4,3\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{15} \geq a_{16} \qquad (41)$$

$$(\langle 11,9,3,2\rangle - \langle 11,10,3,1\rangle + \langle 14,11\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{16} \geq a_{15}. \text{Hence } a_{15} = a_{16} \qquad (42)$$

$$(\langle 10,6,5,3,1\rangle^* - \langle 9,6,5,3,2\rangle^* + \langle 10,9,3,2,1\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{21} \geq a_{23} \qquad (43)$$

$$(\langle 9,6,5,3,2\rangle^* - \langle 10,6,5,3,1\rangle^* + \langle 11,6,5,3\rangle) \uparrow^{(4,8)} S_{26} \Rightarrow a_{23} \geq a_{21}. \text{ Hence } a_{21} = a_{23}, \qquad (44)$$

then deg$Y_1, Y_2 \equiv 0 \; mod \; 11^2$ only when $Y_1 + Y_2 = n_1k_{13} + n_2k_{15}$, $n_1 = 0, n_2 = 1$ or $n_1 = 1, n_2 \in \{0,1\}$. So $k_{13} = d_{31} + d_{32}$.

For k_{14}, let $a_{24} = 1$. By restricting and inducing then it splits to d_{37} and d_{38}. For k_{15} let $a_{23} \in \{1,2\}$ and since

$$(\langle 10,9,3,2,1\rangle^* - \langle 7,6,5,4,3\rangle^*) \uparrow^{(4,8)} S_{26} \Rightarrow a_{24} = 0. \qquad (45)$$

So it's splits to d_{39} and d_{40}. Finally we have 206 columns, and then k_7 must divided into d_{17} and d_{18}. The decomposition matrix is shown in Table (3).

References

[1] G. D. James, A. Kerber, The representation theory of the symmetric group. Mass, Addison-Wesley(1981)

[2] A.O. Morris, A. K. Yaseen, Decomposition matrices for spin characters of symmetric group. Proc. of Royal society of Edinburgh 108A (1988)145-164.

[3] M. M. Jawad, On Brauer Trees and Decomposition Matrices of Spin Characters of S_n Modulo 13,11(2018).

[4] A.O. Morris, The spin representation of the symmetric group. proc. London Math. Soc (3)12 (1962) 55-76
[5] B.M. Puttaswamaiah, J.D. Dixon, Modular representation of finite groups. Academic Press, J.london Math. Soc 15 (1977) 445-455.
[6] A. H. Jassim: 7-Modular Character of The Covering group $\bar{S}_{23}$. Journal of Basrah Researches ((Sciences)) V 43. N. 1 A (2017) 108-129.
[7] I. Schur, Uber die Darstellung der symmetrischen und der alternierenden gruppe durch gebrochene lineare subtituttionen. j.Reine ang.Math 139(1911) 155-250.
[8] J. F. Humphreys, Projective modular representations of finite groups. J. London Math. Society2.16(1977)51-66.

مصفوفات التجزئة للمشخصات الأسقاطية للزمرة التناظرية S_{26}, $p = 11$

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المستخلص

في هذا البحث لقد وجدنا مصفوفات التجزئة للمشخصات الأسقاطية للزمرة التناظرية S_{26}, $p = 11$ والتي هي علاقة بين المشخصات الاعتيادية والمعيارية