# Fano and Dicke Effects in Parallel-Coupled Quantum Dots Embedded Between Two Leads

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**Doi:10.29072/basjs.20230306**

**ARTICLE INFO**

**Keywords**

Green-function, Dicke effect, Fano effect, equation of motion

**ABSTRACT**

In this article, electronic transport through a double quantum dot attached in parallel to a metal leads and with the presence of a magnetic flux is studied with the help of the Green-function formalism and the equation of motion approach. A detail formula is obtained for the density of states and the conductance. The Fano and Dicke effects in the conductance spectra of the double quantum dots system DQD is found and controlled by tuning the coupling (Γ) of the two dots to right (left) Leads and by changing the external magnetic flux.

**Received 11 Feb 2023; Received in revised form 22 July 2023; Accepted 18 Sep 2023, Published 31, Dec 2023**

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## 1. Introduction

The interference effects in the electron transport through multiple quantum dots system are the subject of permanent interest, since the systems that composed of two or more quantum dots coupled to metallic leads are the suitable systems where the interference effects are clearly visible. The most interesting among the various quantum interference phenomenon that observed experimentally in quantum dots QDs system are the Fano and Dicke effects. For instance, J. Gores and others[1] used Fano effect in single electron transistor and they observed asymmetric Fano resonances in the conductance of a single-electron transistor resulting from interference between a resonant and a non-resonant path through the system. The resonant component shows all the features of single-electron transistor, but the non-resonant path is unclear. While, Bogdan and others[2] studied electronic transport through a quantum dot strongly coupled to electrodes within a model with two conduction channels. It was shown that interference of transmitted waves through both channels lead to Fano resonance. Moreover, Piotr Trocha and Józef Barnaś[3] analyzed theoretically spin-dependent transport through two coupled single-level quantum dots attached to ferromagnetic leads by utilization the Green function technique. The numerical analysis was focused on the Fano anti-resonance interference and Coulomb interaction effects. They found that the presence of Fano anti-resonance depends on the sign of the non-diagonal coupling elements. While Chandra Sekhar [4] and others used Dicke effect to make CNOT gate and single-qubit gate in quantum computer more stable and more efficiently by using concise realizations. In this work, we study electronic transport through a double-quantum dot molecule attached to two leads, in a transition from a connection in series to a completely symmetrical parallel configuration by changing magnetic flux, for intermediate values of the flux (semi-integer multiples of a quantum of flux) double quantum dot behaves as series configuration and when the flux is close to integer multiples the double quantum dots behaves as parallel configuration, the density of states shows an narrow and a broad peak at the energies of the molecular states, associated with Fano line shapes in the conductance.

## 2. Model

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We consider two single-level quantum dots, coupled to left and right leads in the way illustrated in Fig. 1. The single Hamiltonian can be extended to describe the two quantum dots in parallel, the electronic Hamiltonian can be described by the Anderson model[5], taking in this model all the coupling interactions between the two dots and between the dots and the two leads. According to this model, the system can be described by the following Hamiltonian:

$$H = H_M + H_L + H_I \tag{1}$$

Where $H_M$ represents the Hamiltonian of the double quantum dots and described by:

$$H_M = \sum_{i=1}^{2} \varepsilon_i c_i^\dagger c_i - t_c\left(c_2^\dagger c_1 + c_1^\dagger c_2\right) \tag{2}$$

Where $t_c$ is the tunneling interaction between two dots, $\varepsilon_i$ is the quantum dots energy level, $c_i^\dagger c_i$ are creation and annihilation operator of the quantum dots energy levels.

$H_L$ represents the Hamilton of the two leads;

$$H_L = \sum_{k_\alpha \in \{L,R\}} \varepsilon_{k_\alpha} a_{k_\alpha}^\dagger a_{k_\alpha} \tag{3}$$

$\varepsilon_{k_\alpha}$ is the energy in leads, $a_{k_\alpha}^\dagger a_{k_\alpha}$ are creation and annihilation operator of the electronic state $k$ in leads levels.

$H_I$ is Hamilton of interaction between dots is given by;

$$H_I = \sum_{k_\alpha \in \{L,R\}} V_{1k_\alpha} c_1^\dagger c_{k_\alpha} + h \cdot c + \sum_{k_\alpha \in \{L,R\}} V_{2k_\alpha} c_2^\dagger c_{k_\alpha} + h \cdot c \tag{4}$$

$V_{1k_\alpha}, V_{2k_\alpha}$ are the coupling matrix elements ($i = 1,2$)between dots and leads($\alpha = R, L$).

Green's function (GFs) is a powerful and clever technique to solve many differential equations in classical mechanics, electrodynamics, and even quantum field theory.

Quantum dots systems are nano structures, then the electron transport through QDs is a quantum transport and is a many body non-equilibrium problem, so it is suitable to study such transport by using the non-equilibrium Green function (NEGF) approach[6]. The

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matrix elements represent the retarded GFs of double quantum dots systems defined as[7-9];

$$G_{ij}^{r}(t) = -i\theta(t)\langle\{c_i(t), c_j^{\dagger}(0)\}\rangle \qquad i = j = 1,2 \tag{5}$$

It is possible to write $c_i(t)$ in terms of the time independent operator such as $c_i(0)$

$$c_i(t) = e^{iHt} \cdot c_i(0)e^{-iHt} \tag{6}$$

With $H$ is given in equation (1).

$$\frac{d}{dt}c_i(t) = iHe^{iHt}c_i(0)e^{-iHt} + e^{iHt}c_i(0)e^{-iHt}(-iH) \tag{7}$$

$$\frac{d}{dt}c_i(t) = ie^{iHt}[H, c_i(0)]e^{-iHt} \tag{8}$$

$$= ie^{iHt}Ae^{-iHt} + ie^{iHt}Be^{-iHt} + ie^{iHt}Ce^{-iHt} \tag{9}$$

For simplicity we use (A, B, C)

$$A = [H_M, c_i(0)] = -\varepsilon_i c_i + t_c c_1 + t_c c_2 \tag{10}$$

$$B = [H_L, c_i(0)] = 0 \tag{11}$$

$$C = [H_I, c_i] = -2V_{1L}c_L - 2V_{1R}c_R - 2V_{2L}c_L - 2V_{2R}c_R \tag{12}$$

$$[H, c_i(0)] = -\varepsilon_i c_i + t_c c_1 + t_c c_2 - 2V_{1L}c_L - 2V_{1R}c_R - 2V_{2L}c_L - 2V_{2R}c_R \tag{13}$$

We started our solution by differentiate equation (5) to get;

$$\frac{d}{dt}G_{ij}^{r}(t) = -i\frac{d}{dt}\theta(t)\langle\{c_i(t), c_j^{\dagger}(0)\}\rangle - i\theta(t)\left\langle\left\{\frac{d}{dt}c_i(t), c_j^{\dagger}(0)\right\}\right\rangle \tag{14}$$

$$i\frac{d}{dt}G_{ij}^{r}(t) = \delta(t)\delta_{ij} + e^{iHt}\theta(t)\langle\{[H, c_i(0)], c_j^{\dagger}(0)\}\rangle e^{-iHt} \tag{15}$$

where

$$\frac{d}{dt}\theta(t) = \delta(t), \quad \delta_{ij} = \{c_i(t), c_j^{\dagger}(0)\} \tag{16}$$

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$$ii\frac{d}{dt}\mathrm{G}_{ij}^{r}(t) = \delta(t)\delta_{ij} + \theta(t)e^{iHt}\big(\langle\{[H_M, c_i(0)], c_j^{\dagger}(0)\} + \{[H_I, c_i(0)], c_j^{\dagger}(0)\}\rangle\big)e^{-iHt} \quad (17)$$

Get use of the definitions of A,B and C then;

$$i\frac{d}{dt}\mathrm{G}_{ij}^{r}(t) = \delta(t)\delta_{ij} + \theta(t)e^{iHt}\big(\langle\{-\varepsilon_i c_i + t_c c_1 + t_c c_2, c_j^{\dagger}\} - \{[2V_{1L}c_L + 2V_{1R}c_R + 2V_{2L}c_L + 2V_{2R}c_R, c_i(0)], c_j^{\dagger}\}\rangle\big)e^{-iHt} \quad (18)$$

with some simplifications;

$$i\frac{d}{dt}\mathrm{G}_{ij}^{r}(t) = \delta(t)\delta_{ij} + \varepsilon_i\mathrm{G}_{ij}^{r}(t) - t_c\mathrm{G}_{ij}^{r}(t) + \sum_{k_\alpha} V_{k_\alpha i}\,\mathrm{G}_{k_\alpha j}^{r}(t) \quad (19)$$

where

$$\mathrm{G}_{k_\alpha j}^{r}(t) = -\theta(t)\{c_{k_\alpha}(t), c_j^{\dagger}\} \quad (20)$$

We use Fourier transformation to transfer from time to energy space.

$$i\frac{d}{dt}\mathrm{G}_{ij}^{r}(t) = (\omega + i\eta)\mathrm{G}_{ij}^{r}(\omega) \quad (21)$$

$$(\omega + i\eta)\mathrm{G}_{ij}^{r}(\omega) = \delta_{ij} + (\varepsilon_i - t_c)\mathrm{G}_{ij}^{r}(\omega) + \sum_{k_\alpha} V_{k_\alpha}\,\mathrm{G}_{k_\alpha j}^{r}(\omega) \quad (22)$$

We assume $t_c \to 0$

$$(\omega + i\eta)\mathrm{G}_{ij}^{r}(\omega) = \delta_{ij} + \varepsilon_i\mathrm{G}_{ij}^{r}(\omega) + \sum_{k_\alpha} V_{k_\alpha}\,\mathrm{G}_{k_\alpha j}^{r}(\omega) \quad (23)$$

$$\mathrm{G}_{ij}^{r}(\omega) = \frac{\delta_{ij}}{(\omega + i\eta - \varepsilon_i)} + \frac{\sum_{kr} V_{k_\alpha i}\,\mathrm{G}_{k_\alpha j}^{r}(\omega)}{(\omega + i\eta - \varepsilon_i)} \quad (24)$$

If we define that $g_i^R(\omega) = \frac{\delta_{ij}}{(\omega + i\eta - \varepsilon_i)}$. Then the retarded Green function will be:

$$\mathrm{G}_{ij}^{r}(\omega) = \delta_{ij}g_i^R(\omega) + g_i^R(\omega)\sum_{k_\alpha} V_{k_\alpha i}\,\mathrm{G}_{k_\alpha j}^{r}(\omega) \quad (25)$$

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Following the same steps, we also can find Green function for leads:

$$i\frac{d}{dt}\mathrm{G}^{r}_{k_\alpha j}(t) = \varepsilon_{k_\alpha}\mathrm{G}^{r}_{k_\alpha j}(t) + \sum_{k_\alpha} V_{k_\alpha j}\,\mathrm{G}^{r}_{ij}(t) \tag{26}$$

And Green function for leads is equal to:

$$\mathrm{G}^{r}_{k_\alpha j}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{G}^{r}_{k_\alpha j}(t)e^{i(\omega+i\eta)t}dt \tag{27}$$

Use Fourier transformation again;

$$i\frac{d}{dt}\mathrm{G}^{r}_{k_\alpha j}(t) = (\omega+i\eta)\mathrm{G}^{r}_{k_\alpha j}(\omega) \tag{28}$$

Using equation (26,28) to get:

$$i\mathrm{G}^{r}_{k_\alpha j}(\omega) = \frac{\sum_{k_\alpha} V_{k_\alpha j}\,\mathrm{G}^{r}_{ij}(\omega)}{\left(\omega+i\eta-\varepsilon_{k_\alpha}\right)} \tag{29}$$

Or

$$\mathrm{G}^{r}_{k_\alpha j}(\omega) = g^{R}_{r}(\omega)\sum_{k_\alpha} V_{k_\alpha j}\,\mathrm{G}^{r}_{ij}(\omega) \tag{30}$$

For

**1.** $\alpha = L,R \quad , i = 1 \ , \ j = 1$

$$(\omega+i\eta)\mathrm{G}^{r}_{11}(\omega) = 1 + \varepsilon_1\mathrm{G}^{r}_{11} + \sum_{k_\alpha}(V_{kL1}\mathrm{G}^{r}_{kL1} + V_{kR1}\mathrm{G}^{r}_{kR1}) \tag{31}$$

Also for $\mathrm{G}^{r}_{k_\alpha j}(\omega)$

$$\mathrm{G}^{r}_{k_\alpha j}(\omega) = \frac{\sum_i V_{k_\alpha i}\,\mathrm{G}^{r}_{ij}(\omega)}{(\omega+i\eta-\varepsilon_{kr})} \tag{32}$$

We use $j = 1 \quad , i = 1{,}2 \ , \ \alpha = L$

$$\mathrm{G}^{r}_{kL1}(\omega) = -\frac{V_{L1}\mathrm{G}_{11} + V_{L2}\mathrm{G}_{21}}{(\varepsilon_{kL}-\omega-i\eta)} \tag{33}$$

$$\mathrm{G}^{r}_{kR1}(\omega) = -\frac{V_{R1}\mathrm{G}_{11} + V_{R2}\mathrm{G}_{21}}{(\varepsilon_{kR}-\omega-i\eta)} \tag{34}$$

Put equations (29,31) in equation 28:

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$$(\omega + i\eta)\mathrm{G}_{11}^{r}(\omega) = 1 + \varepsilon_1 \mathrm{G}_{11}^{r} + \sum_{k_\alpha}\left[V_{kL1}\left(-\frac{V_{L1}\mathrm{G}_{11} + V_{L2}\mathrm{G}_{21}}{(\varepsilon_{kL} - \omega - i\eta)}\right) + V_{kR1}\left(-\frac{V_{R1}\mathrm{G}_{11} + V_{R2}\mathrm{G}_{21}}{(\varepsilon_{kR} - \omega - i\eta)}\right)\right] \quad (35)$$

$$(\omega + i\eta)\mathrm{G}_{11}^{r}(\omega) = 1 + \varepsilon_1 \mathrm{G}_{11}^{r} + (-V_{L1}^{*}V_{L1}A_L\mathrm{G}_{11}) + (-V_{L1}^{*}V_{L2}A_L\mathrm{G}_{21}) + (-V_{R1}^{*}V_{R1}A_R\mathrm{G}_{11}) + (-V_{R1}^{*}V_{R2}A_R\mathrm{G}_{21}) \quad (36)$$

Introducing the magnetic flux effect ($\phi$) on the two leads in the way below;

$$\begin{aligned} V_{L1} &= V_L e^{-i\phi/4} & V_{L1}^{*} &= V_L e^{i\phi/4} \\ V_{L2} &= V_L e^{i\phi/4} & V_{L2}^{*} &= V_L e^{-i\phi/4} \\ V_{R1} &= V_R e^{i\phi/4} & V_{R1}^{*} &= V_R e^{-i\phi/4} \\ V_{R2} &= V_R e^{-i\phi/4} & V_{R2}^{*} &= V_R e^{i\phi/4} \end{aligned} \quad (37)$$

And

$$A_\alpha = \sum_k \frac{1}{(\varepsilon_{k\alpha} - \omega - i\eta)} \quad (38)$$

So, equation (36) becomes:

$$(\omega + i\eta)\mathrm{G}_{11}(\omega) = 1 + \varepsilon_1 \mathrm{G}_{11}(\omega) - V_L^2 A_L \mathrm{G}_{11} - V_L^2 A_L \mathrm{G}_{21} e^{i\phi/2} - V_R^2 A_R \mathrm{G}_{11} - V_R^2 A_R \mathrm{G}_{21} e^{-i\phi/2} \quad (39)$$

The two leads are similar we can put, $V_L^2 = V_R^2 = V^2$ then equation (38) will be reduced to:

$$(\omega + i\eta)\mathrm{G}_{11}(\omega) = 1 + \mathrm{G}_{11}[\varepsilon_1 - V^2(A_L + A_L)] - V^2\mathrm{G}_{21}\left[A_L e^{i\phi/2} + A_R e^{-i\phi/2}\right] \quad (40)$$

**2.** $\alpha = L, R \quad , i = 2 \ , \ j = 1$

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$$(\omega + i\eta)\mathrm{G}_{21}^{r}(\omega) = 0 + \varepsilon_2 \mathrm{G}_{21}^{r}(\omega) + \sum_{k_\alpha}\left[V_{kL2}\left(-\frac{V_{L1}\mathrm{G}_{11} + V_{L2}\mathrm{G}_{21}}{(\varepsilon_{kL} - \omega - i\eta)}\right) + V_{kR1}\left(-\frac{V_{R1}\mathrm{G}_{11} + V_{R2}\mathrm{G}_{21}}{(\varepsilon_{kR} - \omega - i\eta)}\right)\right] \quad (41)$$

And get use of equations (37) to find the element $\mathrm{G}_{21}^{r}$

$$\mathrm{G}_{21} = -V^2 \frac{\mathrm{G}_{11}\left[A_L e^{-i\phi/2} + A_R e^{i\phi/2}\right]}{[(\omega + i\eta) - \varepsilon_2 + V^2(A_L + A_L)]} \quad (42)$$

Put equation (42) in equation (40), and after some work we get $\mathrm{G}_{11}$, the first element in the Green matrix.

$$\mathrm{G}_{11} = \left[\omega + i\eta - \varepsilon_1 + V^2(A_L + A_L) - V^4\left(\frac{A_L^2 + A_R^2 + A_L A_R\left(e^{i\phi} + e^{-i\phi}\right)}{[(\omega + i\eta) - \varepsilon_2 + V^2(A_L + A_L)]}\right)\right]^{-1} \quad (43)$$

Using equation (34)

$$\mathrm{G}_{11} = \frac{1 - V^2\mathrm{G}_{21}\left[A_L e^{i\phi/2} + A_R e^{-i\phi/2}\right]}{\left(\omega + i\eta - \varepsilon_1 + V^2(A_L + A_L)\right)} \quad (44)$$

From equation (42), $\mathrm{G}_{21}$ will have the form

$$\mathrm{G}_{21} = \left[\frac{[(\omega + i\eta) - \varepsilon_2 + V^2(A_L + A_L)]\left(\omega + i\eta - \varepsilon_1 + V^2(A_L + A_L)\right)}{-V^2\left[A_L e^{-\frac{i\phi}{2}} + A_R e^{\frac{i\phi}{2}}\right]} + V^2\left[A_L e^{-\frac{i\phi}{2}} + A_R e^{\frac{i\phi}{2}}\right]\right]^{-1} \quad (45)$$

For simplicity we define:

$$A_L^2 = A_R^2, \qquad \eta = 0$$

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$$V^2[A_L + A_R] = V^2 \sum_{k_\alpha}^{1} \frac{1}{\varepsilon_{kr} - \omega - i\eta} \cong i\frac{\Gamma}{2} \tag{46}$$

Where $\Gamma^L, \Gamma^R$ describes the tunneling coupling of the two quantum dots to left and right leads and given by the below equations[10];

$$\Gamma^{\mathrm{R}} = \frac{\Gamma}{2}\begin{bmatrix} 1 & e^{-\frac{i\phi}{2}} \\ e^{\frac{i\phi}{2}} & 1 \end{bmatrix} \tag{47}$$

$$\Gamma^{\mathrm{L}} = \frac{\Gamma}{2}\begin{bmatrix} 1 & e^{\frac{i\phi}{2}} \\ e^{-\frac{i\phi}{2}} & 1 \end{bmatrix} \tag{48}$$

From equations (44,46) we can get $\mathrm{G}_{11}$;

$$\mathrm{G}_{11} = \frac{\left(\omega - \varepsilon_2 + i\frac{\Gamma}{2}\right)}{\left(\omega - \varepsilon_1 + i\frac{\Gamma}{2}\right)\left(\omega - \varepsilon_2 + i\frac{\Gamma}{2}\right) + \frac{\Gamma^2}{4}\cos^2\frac{\phi}{2}} \tag{49}$$

In the same way can find $\mathrm{G}_{21}, \mathrm{G}_{12}, \mathrm{G}_{22}$, such as;

$$\mathrm{G}_{21} = \frac{-i\frac{\Gamma}{2}\cos\frac{\phi}{2}}{\left(\omega - \varepsilon_1 + i\frac{\Gamma}{2}\right)\left(\omega - \varepsilon_2 + i\frac{\Gamma}{2}\right) + \frac{\Gamma^2}{4}\cos^2\frac{\phi}{2}} \tag{50}$$

$$\mathrm{G}_{22} = \frac{\left(\omega - \varepsilon_1 + i\frac{\Gamma}{2}\right)}{\left(\omega - \varepsilon_1 + i\frac{\Gamma}{2}\right)\left(\omega - \varepsilon_2 + i\frac{\Gamma}{2}\right) + \frac{\Gamma^2}{4}\cos^2\frac{\phi}{2}} \tag{51}$$

$$\mathrm{G}_{22} = \frac{\left(\omega - \varepsilon_1 + i\frac{\Gamma}{2}\right)}{\left(\omega - \varepsilon_1 + i\frac{\Gamma}{2}\right)\left(\omega - \varepsilon_2 + i\frac{\Gamma}{2}\right) + \frac{\Gamma^2}{4}\cos^2\frac{\phi}{2}} \tag{52}$$

$$\mathrm{G}^{\mathrm{R}}(\omega) = \begin{bmatrix} \mathrm{G}_{11} & \mathrm{G}_{12} \\ \mathrm{G}_{21} & \mathrm{G}_{22} \end{bmatrix} \tag{53}$$

The retarded green function matrix will be:

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$$\mathrm{G}^{\mathrm{R}}(\omega) = \frac{1}{\left(\omega-\varepsilon_1+i\frac{\Gamma}{2}\right)\left(\omega-\varepsilon_2+i\frac{\Gamma}{2}\right)+\frac{\Gamma^2}{4}\cos^2\frac{\phi}{2}}\begin{bmatrix}\left(\omega-\varepsilon_2+i\frac{\Gamma}{2}\right) & -i\frac{\Gamma}{2}\cos\frac{\phi}{2}\\ -i\frac{\Gamma}{2}\cos\frac{\phi}{2} & \left(\omega-\varepsilon_1+i\frac{\Gamma}{2}\right)\end{bmatrix} \quad (5$$

With the advanced green function $\mathrm{G}^{\mathrm{A}}(\omega)$ be;

$$\mathrm{G}^{\mathrm{A}}(\omega) = \frac{1}{\left(\omega-\varepsilon_1+i\frac{\Gamma}{2}\right)\left(\omega-\varepsilon_2+i\frac{\Gamma}{2}\right)+\frac{\Gamma^2}{4}\cos^2\frac{\phi}{2}}\begin{bmatrix}\left(\omega-\varepsilon_2-i\frac{\Gamma}{2}\right) & i\frac{\Gamma}{2}\cos\frac{\phi}{2}\\ i\frac{\Gamma}{2}\cos\frac{\phi}{2} & \left(\omega-\varepsilon_1-i\frac{\Gamma}{2}\right)\end{bmatrix} \quad (5$$

## 3. Transmission Rate Calculation

To find the transmission rate we use equation[8-12];

$$T(\omega) = Tr\{\mathrm{G}^{\mathrm{A}}(\omega)\Gamma^{\mathrm{R}}\mathrm{G}^{\mathrm{R}}(\omega)\Gamma^{\mathrm{L}}\} \quad (56)$$

$$T(\omega) = Tr\left\{\begin{bmatrix}\mathrm{G}_{11}^{A} & \mathrm{G}_{12}^{A}\\ \mathrm{G}_{21}^{A} & \mathrm{G}_{22}^{A}\end{bmatrix}\begin{bmatrix}\Gamma_{11}^{R} & \Gamma_{12}^{R}\\ \Gamma_{21}^{R} & \Gamma_{22}^{R}\end{bmatrix}\begin{bmatrix}\mathrm{G}_{11}^{R} & \mathrm{G}_{12}^{R}\\ \mathrm{G}_{21}^{R} & \mathrm{G}_{22}^{R}\end{bmatrix}\begin{bmatrix}\Gamma_{11}^{L} & \Gamma_{12}^{L}\\ \Gamma_{21}^{L} & \Gamma_{22}^{L}\end{bmatrix}\right\} \quad (57)$$

$$T(\omega) = \mathrm{T}_{11} + \mathrm{T}_{22} \quad (58)$$

We solve equation (57) with all the matrix element defined and the we get $\mathrm{T}_{ij}$ such as;

$$\begin{aligned}\mathrm{T}_{11} = \Bigg[&(\omega-\varepsilon_2)^2 - \frac{\Gamma^2}{4} - i\Gamma\cos\frac{\phi}{2}e^{-\frac{i\phi}{2}}\left(\omega-\varepsilon_2-i\frac{\Gamma}{2}\right)\\ &+ i\frac{\Gamma}{2}\cos\frac{\phi}{2}e^{\frac{i\phi}{2}}\left(\omega-\varepsilon_2+i\frac{\Gamma}{2}\right)+\frac{\Gamma^2}{2}\cos^2\frac{\phi}{2}\\ &+\left(\omega-\varepsilon_2-i\frac{\Gamma}{2}\right)\left(\omega-\varepsilon_1+i\frac{\Gamma}{2}\right)e^{-i\phi}\\ &+ i\frac{\Gamma}{2}\cos\frac{\phi}{2}\left(\omega-\varepsilon_1-i\frac{\Gamma}{2}\right)e^{-\frac{i\phi}{2}}\Bigg]\end{aligned} \quad (59)$$

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$$\begin{aligned} \mathrm{T}_{22} = \Bigg[ & (\omega-\varepsilon_1)^2 + \frac{\Gamma^2}{4} + \frac{\Gamma^2}{2}cos^2\frac{\phi}{2} + i\frac{\Gamma}{2}cos\frac{\phi}{2}e^{\frac{i\phi}{2}}\left(\omega-\varepsilon_2+i\frac{\Gamma}{2}\right) \\ & + \left(\omega-\varepsilon_2-i\frac{\Gamma}{2}\right)\left(\omega-\varepsilon_1+i\frac{\Gamma}{2}\right)e^{i\phi} \\ & - i\Gamma\, cos\frac{\phi}{2}e^{\frac{i\phi}{2}}\left(\omega-\varepsilon_2-i\frac{\Gamma}{2}\right) \\ & + i\frac{\Gamma}{2}cos\frac{\phi}{2}\left(\omega-\varepsilon_1+i\frac{\Gamma}{2}\right)e^{-\frac{i\phi}{2}}\Bigg] \end{aligned} \tag{60}$$

Finally $T(\omega)$ will be;

$$T(\omega) = \frac{\Gamma^2\left(4\cos^2\frac{\phi}{2}(\omega-\overline{\varepsilon})^2 + 4\left(\frac{\Delta\varepsilon}{2}\right)^2\sin^2\frac{\phi}{2}\right)}{\left[(\omega-\overline{\varepsilon})^2 - \left(\frac{\Delta\varepsilon}{2}\right)^2 - \frac{\Gamma^4}{4}\sin^2\frac{\phi}{2}\right]^2 + [\Gamma(\omega-\overline{\varepsilon})]^2} \tag{61}$$

Where $\overline{\varepsilon} = \frac{\varepsilon_1+\varepsilon_2}{2}$, $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$

## 4. Density of State Calculation

We get use of the diagonal elements of the Green's function matrix to calculate the spectra densities $A_{\pm}$, then the summing overall $\pm$ states we obtain $\rho(\varepsilon)$ the density of states of the QDs where[7, 9];

$$\rho(\varepsilon) = \sum_{\sigma=-,+} A_\sigma \tag{62}$$

$$A_- = \frac{1}{\pi\Lambda}\cos^2\left(\frac{\phi}{4}\right)\tilde{\Gamma}\left[(t_c-\varepsilon)^2 + 4\Gamma^L\Gamma^R\sin^4\left(\frac{\phi}{4}\right)\right] \tag{63}$$

$$A_+ = \frac{1}{\pi\Lambda}\sin^2\left(\frac{\phi}{4}\right)\tilde{\Gamma}\left[(t_c+\varepsilon)^2 + 4\Gamma^L\Gamma^R\cos^4\left(\frac{\phi}{4}\right)\right] \tag{64}$$

$$\Lambda = \tilde{\Gamma}^2\left[\varepsilon - t_c\cos\left(\frac{\phi}{2}\right)\right] + \left[(t_c+\varepsilon)(t_c-\varepsilon) + \Gamma^L\Gamma^R\sin^2\left(\frac{\phi}{2}\right)\right]^2 \tag{65}$$

$$\begin{aligned} \rho(\varepsilon) = & \frac{1}{\pi\Lambda}\cos^2\left(\frac{\phi}{4}\right)\tilde{\Gamma}\left[(t_c-\varepsilon)^2 + 4\Gamma^L\Gamma^R\sin^4\left(\frac{\phi}{4}\right)\right] \\ & + \frac{1}{\pi\Lambda}\sin^2\left(\frac{\phi}{4}\right)\tilde{\Gamma}\left[(t_c+\varepsilon)^2 + 4\Gamma^L\Gamma^R\cos^4\left(\frac{\phi}{4}\right)\right] \end{aligned} \tag{66}$$

Finally, the density of states for quantum dot molecule can be calculated from:

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$$\rho(\varepsilon) = \frac{\tilde{\Gamma}\,\Gamma^{L}\Gamma^{R}\cos^{2}\left(\frac{\phi}{4}\right)\sin^{2}\left(\frac{\phi}{4}\right) + \tilde{\Gamma}\left[(t_c-\varepsilon)^{2}\cos^{2}\left(\frac{\phi}{4}\right) + (t_c+\varepsilon)^{2}\sin^{2}\left(\frac{\phi}{4}\right)\right]}{\pi\Lambda} \quad (67)$$

With $\tilde{\Gamma} = \Gamma^{L} + \Gamma^{R}$.

## 5. Result and Discussion

In this section, we studied the numerical calculation (using Matlab simulation) for the density of states and conductance at zero temperature of two coupled quantum dots connected symmetrically to leads in a parallel configuration under the effect of a magnetic flux. All our calculations are performed for a weak coupling $t_c = 0$ regime and for strong coupling, where $t_c = \Gamma, (t_c \leq \Gamma_{R,L})$, equation (67) was applied to get the density of states as a function of the energy ($\varepsilon$).

Fig.2a represent the density of states of the DQD for ($t_c = 0$) and Fig.2 (b,c,d,e,f) for ($t_c = \Gamma$) with ($\Gamma_L = \Gamma_R$) and ($\Gamma = \Gamma_L + \Gamma_R$) and for different magnetic flux.

A special case for the density of state distribution when $t_c = 0$ and $\phi = 0$ (as shown in Fig.2a) the distributions that represented Lorentzians shape have peak centered at $\varepsilon = 0$, while for $\phi = \pi$ and $t_c = \Gamma$ (Fig.2b) the distributions are identical and represented by the superposition of two Lorentzians separated by $2t_c$. Also we have the same behaviors in Fig.2 (c,d) for $\phi = 0.5\pi, 1.5\pi$ and $t_c = \Gamma$ but the peaks are unidentical.

The Dicke effect in parallel configuration occurs mostly in symmetrical case where ($\Gamma_L = \Gamma_R$) and whenever the magnetic close to the integer of flux quanta ($\phi \cong 2\pi n, n = integer$).

By changing the value of magnetic flux to be $\phi \cong 2\pi n$, two peaks are exist in fig.2(e,f), and we begin to see a narrowing of one distribution and a widening of the other one.

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The figures show two peaks, one is a wide states and the other is a narrow states. These states are interchanged their positions for different $\phi$, and it is obvious that the peaks in these figures are lying on ($\varepsilon = -2,2$) depends on the value of tunneling coupling ($t_c$).

In particular, when the magnetic flux is near an integer number of flux quanta, the system is in the Dicke regime. From the densities of states, it can be deduced that the antibonding (bonding) state becomes progressively localized as the magnetic flux tends to an integer number of flux quanta. When the magnetic flux is exactly an integer, tunneling through the antibonding (bonding) state is totally suppressed and the bonding (antibonding) state is the only participating state in the transmission. The controlling of the decoherence processes with the magnetic field exhibited by the present system may have applications in quantum computing.

To study the transport through the quantum dot system, the below equation is required for the conductance at zero temperature[8, 9].

$$G(\varepsilon) = \frac{2e^2}{h} T(\varepsilon) \tag{67}$$

The conductance spectrum is composed of Fano line shapes at the bonding and antibonding energies, and vice versa, depending on whether this number is even or odd, with their line broadenings controlled by the magnetic flux. The narrowing (broadening) of a line in the conductance can be interpreted as an increase (decrease) in the lifetime of the corresponding molecular state.

So, Fig (3) demonstrates the development of conductance with changing the magnetic flux, these figures shows two peaks for each value of magnetic flux, these peaks are different in width and height.

From these figs and for ($\phi = 0,0.1\pi, 1.9\pi, \pi$) two peaks are appear in the conductance spectra this behavior is similar to that of the density of states.

In fig (3a) with ($\phi = 0$) no Fano peak exists. However if $\phi$ is increased from ($\phi = 0$), the channel connecting the decoupling state and the leads is opened and Fano interference exists[13].

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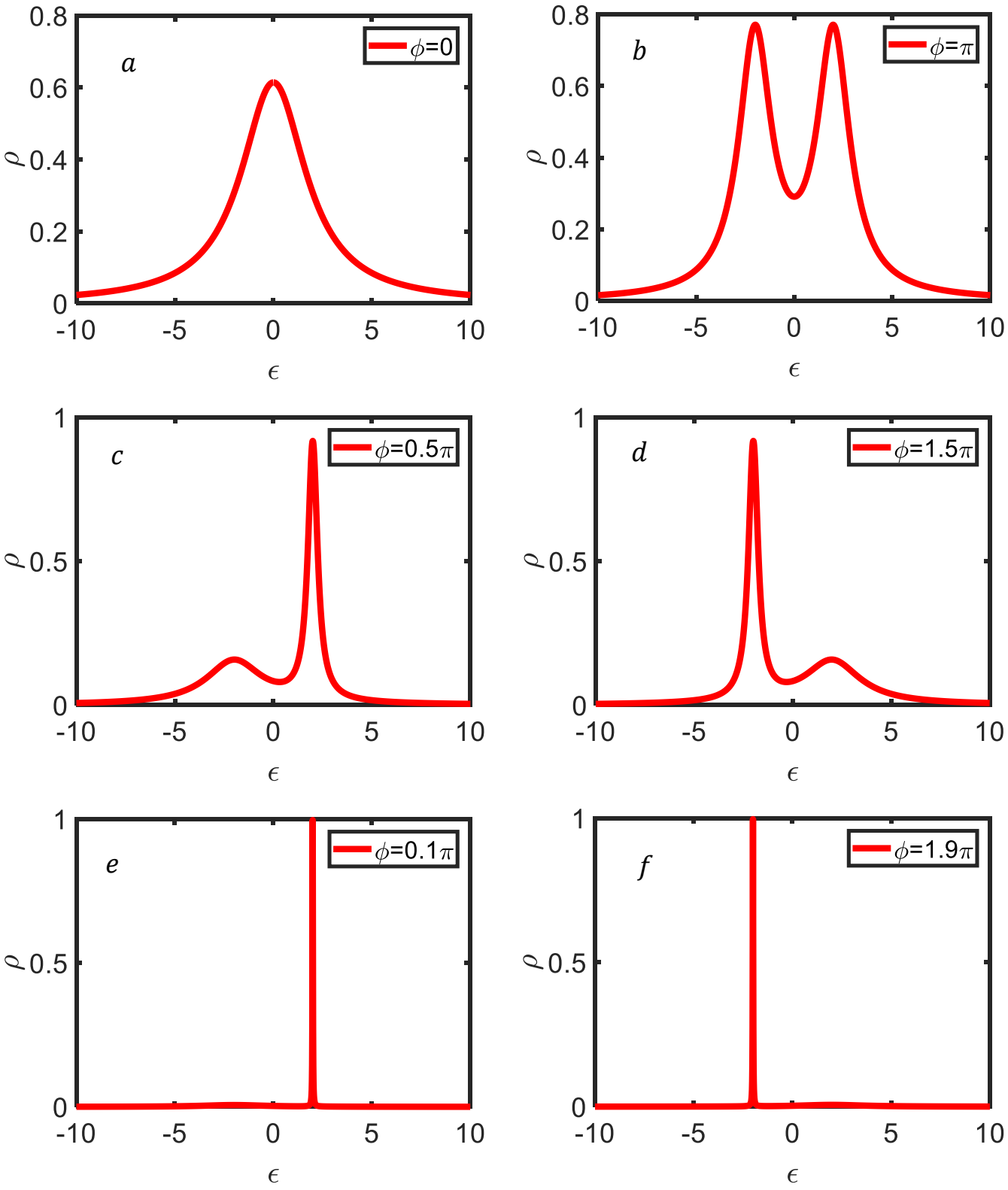


**Figure 1: Density of states (ρ) as a function of energy (ε), with $\Gamma_L = \Gamma_R = \Gamma_0 = 1, \Gamma = \Gamma_L + \Gamma_R$ for (a) $t_c = 0$ and (b,c,d,e,f) $t_c = \Gamma$, with different magnetic flux as shown in figure.**

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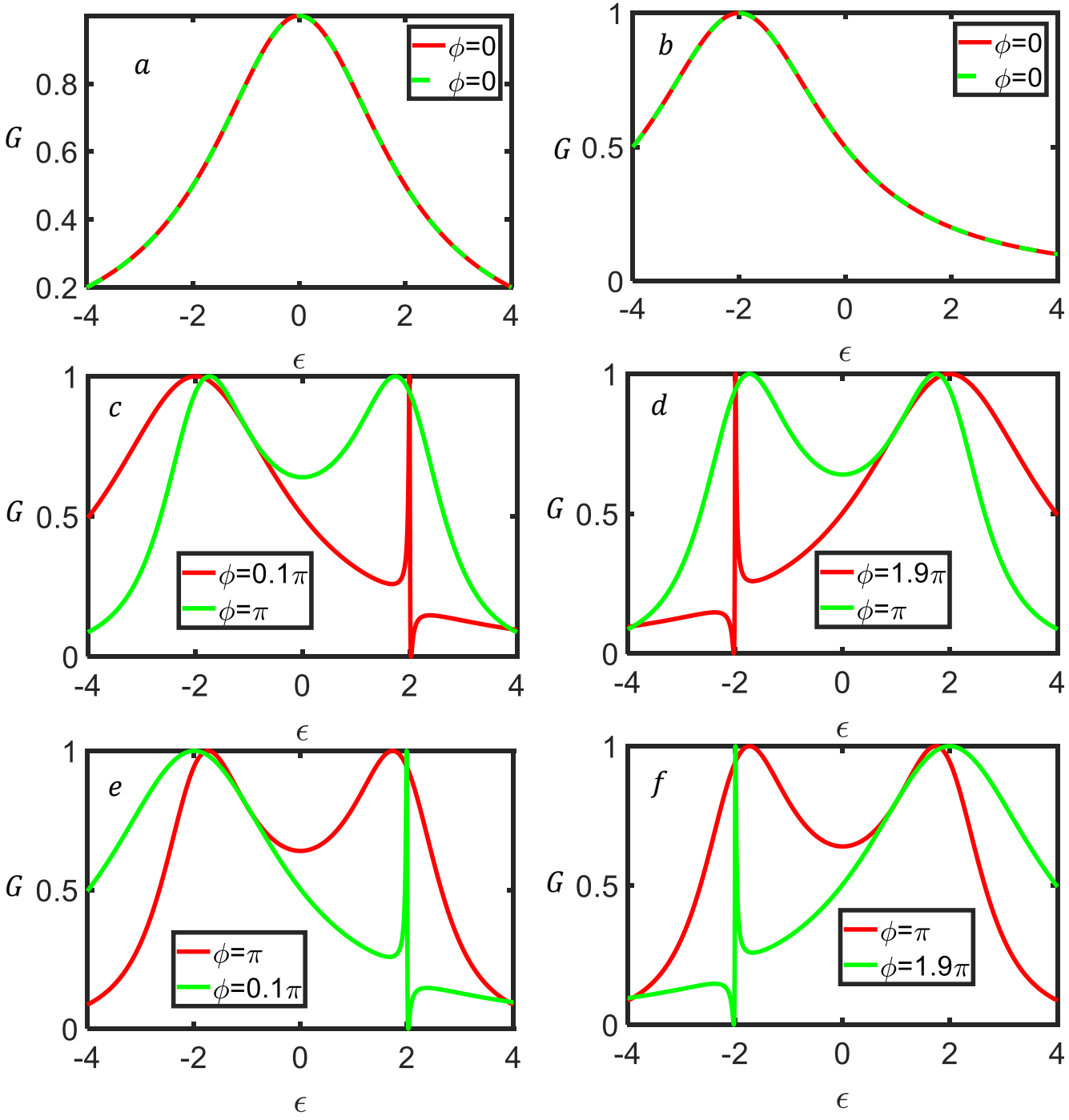


**Figure 2: conductance (g) as a function of energy (ε), with $\Gamma_L = \Gamma_R = \Gamma_0 = 1, \Gamma = \Gamma_L + \Gamma_R$ for (a) $t_c = 0$ and (b,c,d,e,f) $t_c = \Gamma$, with different magnetic flux as shown in figure.**

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**References**

[1] J. D. Göres, D. G. Gordon, S. M. Heemeyer, D. M. Mahalu, Fano resonances in electronic transport through a single-electron transistor. Phys. Rev. B, 62(2000) 2188. https://doi.org/10.1103/PhysRevB.62.2188

[2] B. R. Bulka, & P. R. Stefański, Fano and Kondo resonance in electronic current through nanodevices. Phys. Rev. Lett, 86(2001), 5128. https://doi.org/10.1103/PhysRevLett.86.5128

[3] P. A. Trocha, J. E. Barnaś, Quantum interference and Coulomb correlation effects in spin-polarized transport through two coupled quantum dots. Phys. Rev. B, 76(2007) 165432. https://doi.org/10.1103/PhysRevLett.86.5128

[4] L. M. Wei, L. S. Liu, S . X. Lei, Quantum computation with two-level trapped cold ions beyond Lamb-Dicke limit. Phys. Rev. A, 65(2002) 062316. https://doi.org/10.1103/PhysRevA.65.062316

[5] M. A. Najdi, J. M. AL-Mukh, H. A. Jassem. "Heat current across double quantum dots in series coupled to ferromagnetic leads in antiparallel configuration within weak interdot coupling regime, J. Comput. Electron, 20 (2021)2403-2410 https://doi.org/10.1007/s10825-021-01778-5

[6] L. N. Remdhan, T. A. Salman. Tunneling magnetoresistance calculation for double quantum dot connected in parallel shape to ferromagnetic Leads, J Kufa-Phys., 15 (2023) 69-76, https://doi.org/10.31257/2018/JKP/2023/v15.i01.11428

[7] M. L. De Guevara, M. L. Claro, P.A. Orellana, Ghost Fano resonance in a double quantum dot molecule attached to leads. Phys. Rev. B, 67(2003) 195335. https://doi.org/10.1103/PhysRevB.67.195335

[8] A. K. Yassir, A. H. Hussain, F. A. Ali, Relativistic Dirac Calculations for Electrons Scattering from Mg and Ca Atoms at Different Energies, J Kufa-Phys., 10(2018)84-99, https//dx.doi.org/10.31257/2018/JKP/100212

[9] L. H. Lü, & B. F. Zhu. Tunable Fano effect in parallel-coupled double quantum dot system. Physical Review B, 71(2005), 235320. https://doi.org/10.1103/PhysRevB.70.233315

[10] M. A. Najdi, J. M. AL-Mukh, H. A. Jassem. "Electron tunneling through serially coupled double quantum dots, In IOP Conference Series: Mater. Sci. Eng., 454(2018) 012043, https://doi.org/10.1088/1757-899X/454/1/012043

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[11] M. L. Niklas, A. D. Trottmann, A. B. Donarini, & M. S. Grifoni. Fano stability diagram of a symmetric triple quantum dot. Phys. Rev. B, 95(2017)115133. https://doi.org/10.1103/PhysRevB.95.115133

[12] A. K. Yassir, Differential Cross-Section Elastic Scattering of Electrons By Silver Atoms." Bas. J Sci., 40(2022)389-399, https://doi.10.29072/basjs.20220211

[13] V. M. Apel, P. A. Orellana, M. P. Pacheco, Fano and Dicke effects and spin polarization in a double Rashba-ring system side coupled to a quantum wire, arXiv preprint arXiv, 93 (2008)0803-4014. https://doi.org/10.48550/arXiv.0803.4014

## تأثيري فانو وديكي في نقاط كمية مزدوجة متوازية مدمجة بين قطبين

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**المستخلص**

في هذه المقالة ، تمت دراسة النقل الإلكتروني من خلال نقطتين كميتين متصلة بالتوازي مع اقطاب معدنية مع وجود تدفق مغناطيسي باستخدام دالة كرين ومعادلة الحركة. تم الحصول على صيغة مفصلة لكثافة الحالات والتوصيلة. لاحظنا ظهور تأثيري فانو وديك في أطياف التوصيل لنظام النقاط الكمومية المزدوجة DQD والتحكم فيها عن طريق ضبط الاقتران (Γ) بين النقطتين الكميتن والاقطاب عن طريق تغيير التدفق المغناطيسي الخارجي.

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